(*iii*) $\frac{4}{9}-\frac{3}{7}=\frac{4\times7-3\times9}{63}=\frac{28-27}{63}=\frac{1}{63}$, which is a rational number.

2. Commutative property

(*i*) *Addition is commutative for rational numbers.*

That is, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$

For example, $\frac{1}{3}+\frac{4}{3}=\frac{4}{3}+\frac{1}{3}$

as $\frac{1}{3}+\frac{4}{3}=\frac{1+4}{3}=\frac{5}{3}$

and $\frac{4}{3}+\frac{1}{3}=\frac{4+1}{3}=\frac{5}{3}$

(*ii*) *Subtraction is not commutative for rational numbers.*

For example, $\frac{2}{3}-\frac{5}{4}\neq\frac{5}{4}-\frac{2}{3}$

as $\frac{2}{3}-\frac{5}{4}=\frac{2\times4-5\times3}{12}=\frac{8-15}{12}=\frac{-7}{12}$

and $\frac{5}{4}-\frac{2}{3}=\frac{5\times3-4\times2}{12}=\frac{15-8}{12}=\frac{7}{12};\ \frac{-7}{12}\neq\frac{7}{12}$

3. Associative property

(*i*) *Addition is associative for rational numbers.*

That is, for any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$$

For example, $\frac{-1}{3}+\left[\frac{2}{7}+\left(\frac{-5}{3}\right)\right]=\left[\frac{-1}{3}+\frac{2}{7}\right]+\left(\frac{-5}{3}\right)$

Here, L.H.S. $=\frac{-1}{3}+\left[\frac{2}{7}+\left(\frac{-5}{3}\right)\right]=\frac{-1}{3}+\left[\frac{2\times3+(-5)\times7}{21}\right]$

$=\frac{-1}{3}+\left(\frac{6-35}{21}\right)=\frac{-1}{3}+\frac{(-29)}{21}$

$=\frac{-7+(-29)}{21}=\frac{-7-29}{21}=\frac{-36}{21}$

and R.H.S. $=\left[\frac{-1}{3}+\frac{2}{7}\right]+\left(\frac{-5}{3}\right)=\left(\frac{-7+6}{21}\right)+\left(\frac{-5}{3}\right)$

$=\frac{-1}{21}+\frac{(-5)}{3}=\frac{-1+(-5)\times7}{21}=\frac{-1-35}{21}=\frac{-36}{21}$

So, L.H.S. = R.H.S. [$\because$ Each side $=\frac{-36}{21}$]

$\therefore\ \frac{-1}{3}+\left[\frac{2}{7}+\left(\frac{-5}{3}\right)\right]=\left[\frac{-1}{3}+\frac{2}{7}\right]+\left(\frac{-5}{3}\right)$

(*ii*) *Subtraction is not associative for rational numbers.*

Is $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\left[\frac{-2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$?

Here, L.H.S. $=\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\frac{-2}{3}-\left(\frac{-8-5}{10}\right)$

$=\frac{-2}{3}-\left(\frac{-13}{10}\right)=\frac{-2}{3}+\frac{13}{10}=\frac{-20+39}{30}=\frac{19}{30}$

and $\quad \text{R.H.S.} = \left[\frac{-2}{3} - \left(\frac{-4}{5}\right)\right] - \frac{1}{2} = \left[-\frac{2}{3} + \frac{4}{5}\right] - \frac{1}{2}$

$$= \left(\frac{-10+12}{15}\right) - \frac{1}{2} = \frac{2}{15} - \frac{1}{2} = \frac{4-15}{30} = \frac{-11}{30}$$

But, $\frac{19}{30} \neq \frac{-11}{30}$

$\therefore \quad \frac{-2}{3} - \left[\frac{-4}{5} - \frac{1}{2}\right] \neq \left[\frac{-2}{3} - \left(\frac{-4}{5}\right)\right] - \frac{1}{2}$

4. Existence of additive identity

0 is called the additive identity of rational numbers because the sum of any rational number and 0 is the rational number itself.

That is, for a rational number $\frac{a}{b}$, $\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$

For example, $\quad \frac{5}{8} + 0 = \frac{5}{8} + \frac{0}{8} = \frac{(5+0)}{8} = \frac{5}{8}$

Similarly, $\quad \left(0 + \frac{5}{8}\right) = \frac{5}{8}$

$\therefore \quad \left(\frac{5}{8} + 0\right) = \left(0 + \frac{5}{8}\right) = \frac{5}{8}$

5. Existence of additive inverse

For every rational number $\frac{a}{b}$, there exists a rational number $\frac{-a}{b}$ such that

$$\frac{a}{b} + \left(\frac{-a}{b}\right) = \frac{a + (-a)}{b} = \frac{0}{b} = 0$$

Also, $\frac{a}{b}$ is the additive inverse of $-\frac{a}{b}$.

$$\left(-\frac{a}{b}\right) + \frac{a}{b} = \frac{(-a) + a}{b} = \frac{0}{b} = 0$$

For example, $\frac{5}{8} + \left(\frac{-5}{8}\right) = \frac{5 + (-5)}{8} = \frac{0}{8} = 0$ and $\left(\frac{-5}{8}\right) + \frac{5}{8} = \frac{(-5)+5}{8} = \frac{0}{8} = 0$

Thus, $\frac{5}{8}$ and $\frac{-5}{8}$ are additive inverse of each other.

Solved Examples

There are four options (Q1 to Q5) out of which only one is correct. Choose the correct option.

1. $\frac{-3}{8} + \frac{1}{7} = \frac{1}{7} + \left(\frac{-3}{8}\right)$ *is an example to show that*

(*a*) *addition of rational numbers is commutative.*

(*b*) *rational numbers are closed under subtraction.*

(*c*) *rational numbers are closed under multiplication.*

(*d*) *rational numbers are distributive under addition.*

Sol. We know that for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

So, $\frac{-3}{8} + \frac{1}{7} = \frac{1}{7} + \left(\frac{-3}{8}\right)$ is an example to show that addition of rational numbers is commutative.

Hence, option (*a*) is the correct answer.

2. *The identity element for the addition of rational numbers is*

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2

Sol. 0 is the identity element for the addition of rational numbers.

Hence, option (*a*) is the correct answer.

3. *The additive inverse of* $\frac{21}{31}$ *is*

(*a*) $\frac{21}{31}$ (*b*) $\frac{-21}{31}$ (*c*) $\frac{31}{-21}$ (*d*) –1

Sol. $\frac{-a}{b}$ is the additive inverse of $\frac{a}{b}$

i.e., $\frac{a}{b}+\left(\frac{-a}{b}\right) = \frac{a+(-a)}{b} = \frac{0}{b} = 0$

So, additive inverse of $\frac{21}{31}$ is $\frac{-21}{31}$

Hence, option (*b*) is the correct answer.

4. *The additive inverse of* $\frac{-7}{19}$ *is*

(*a*) $\frac{-7}{19}$ (*b*) $\frac{7}{19}$ (*c*) $\frac{19}{7}$ (*d*) $\frac{-19}{7}$

Sol. The additive inverse of $\frac{-7}{19}$ is $\frac{7}{19}$

as $\frac{-7}{19}+\frac{7}{19} = \frac{-7+7}{19} = \frac{0}{19} = 0$

Hence, option (*b*) is the correct answer.

5. *The numerical expression* $\frac{3}{8}+\frac{(-5)}{7} = \frac{-19}{56}$ *shows that*

(*a*) *rational numbers are closed under addition.*

(*b*) *rational numbers are not closed under addition.*

(*c*) *rational numbers are closed under multiplication.*

(*d*) *addition of rational numbers is not commutative.*

Sol. We know that for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

So, $\frac{3}{8}+\frac{(-5)}{7} = \frac{-19}{56}$ shows, that rational numbers are closed under addition.

Hence, option (*a*) is the correct answer.

6. *Fill in the blanks to make the statements true.*

(*i*) *If* 0 *is added to any rational number* $\frac{a}{b}$*, the sum is again that* ....

(*ii*) $\frac{-17}{21}+\frac{17}{21} = 0$*, shows that the additive inverse of* $\frac{17}{21}$ *is* ....

(*iii*) *Commutativity for rational numbers states that two rational numbers can be added in any* ....

(*iv*) *Addition is associative for rational numbers means if* $\frac{a}{b}$, $\frac{c}{d}$ *and* $\frac{e}{f}$ *are any three rational numbers, then* $\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right) =$ ....

Sol. (*i*) rational number $\frac{a}{b}$ (*ii*) $\frac{-17}{21}$ (*iii*) order (*iv*) $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}$

7. *In each of the following, state whether the statements are true (T) or false (F).*

(*i*) *The additive inverse of* $\frac{1}{2}$ *is* –2.

(*ii*) *If* $\frac{x}{y}$ *is the additive inverse of* $\frac{c}{d}$*, then* $\frac{x}{y}+\frac{c}{d} = 0$.

(*iii*) *For every rational* number x, $x + 1 = x$.

(*iv*) *If* $\frac{x}{y}$ *is the additive inverse of* $\frac{c}{d}$, *then* $\frac{x}{y} - \frac{c}{d} = 0$.

(*v*) *If* $x + y = 0$, *then* $-y$ *is known as the negative of* x, *where* x *and* y *are rational numbers.*

(*vi*) *The negative of the negative of any rational number is the number itself.*

(*vii*) *The negative of 0 does not exist.*

(*viii*) *The negative of 1 is 1 itself.*

(*ix*) *For all rational numbers* x *and* y, $x - y = y - x$.

(*x*) *Rational numbers are closed under addition but not under subtraction.*

Sol. (*i*) As the additive inverse of $\frac{1}{2}$ is $-\frac{1}{2}$, so the given statement is false.

(*ii*) The given statement is true because sum of a rational number and its additive inverse is always zero.

(*iii*) The given statement is false. For example, consider a rational number $\frac{3}{4}$.

Now, $\frac{3}{4} + 1 = \frac{3+4}{4} = \frac{7}{4}$, which shows that for every rational number x, $x + 1 = x$ is a false statement.

(*iv*) If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y} - \frac{c}{d} = 0$, is a false statement.

(*v*) If $x + y = 0$, then $-y$ is known as the negative of x, where x and y are rational numbers, is a false statement because $x + y = 0 \Rightarrow x = -y$.

(*vi*) The negative of the negative of any rational number is the number itself, is a true statement.

(*vii*) The negative of 0 does not exist, is a true statement.

(*viii*) The negative of 1 is 1 itself, is a false statement.

(*ix*) For all rational numbers x and y, $x - y = y - x$, is a false statement because $x - y \neq y - x$.

(*x*) Rational numbers are closed under addition but not under subtraction, is a false statement.

8. (*i*) *Add* $\frac{-3}{4}, \frac{7}{4}, \frac{11}{4}$

(*ii*) *Find the sum of* $\frac{2}{5}, \frac{-3}{7}, \frac{1}{10}, \frac{8}{21}$

Sol. (*i*)
$$\frac{-3}{4} + \frac{7}{4} + \frac{11}{4} = \left(\frac{-3}{4} + \frac{7}{4}\right) + \frac{11}{4} = \left(\frac{-3+7}{4}\right) + \frac{11}{4} = \frac{4}{4} + \frac{11}{4} = \frac{4+11}{4} = \frac{15}{4}$$

(*ii*)
$$\frac{2}{5} + \frac{-3}{7} + \frac{1}{10} + \frac{8}{21} = \left(\frac{2}{5} + \frac{1}{10}\right) + \left(\frac{-3}{7} + \frac{8}{21}\right)$$

[Rearranging the terms do not alter the sum]

$$= \left(\frac{2 \times 2 + 1}{10}\right) + \left(\frac{-3 \times 3 + 8}{21}\right)$$

$$= \frac{5}{10} + \frac{-1}{21} = \frac{5 \times 21 + (-1) \times 10}{210} = \frac{105 - 10}{210} = \frac{95}{210} = \frac{5 \times 19}{5 \times 42} = \frac{19}{42}$$

9. *Rearrange suitably and find the sum* $\frac{2}{7} + \frac{3}{8} + \frac{4}{21} + \frac{9}{16}$

Sol.
$$\frac{2}{7} + \frac{3}{8} + \frac{4}{21} + \frac{9}{16} = \left(\frac{2}{7} + \frac{4}{21}\right) + \left(\frac{3}{8} + \frac{9}{16}\right)$$

[Rearranging the terms do not after the sum]

$$= \left(\frac{2 \times 3}{7 \times 3} + \frac{4}{21}\right) + \left(\frac{3 \times 2}{8 \times 2} + \frac{9}{16}\right)$$

$$= \left(\frac{6}{21}+\frac{4}{21}\right)+\left(\frac{6}{16}+\frac{9}{16}\right)$$

$$= \left(\frac{6+4}{21}\right)+\left(\frac{6+9}{16}\right) = \frac{10}{21}+\frac{15}{16}$$

$$= \frac{10\times16}{21\times16}+\frac{15\times21}{16\times21} = \frac{160}{336}+\frac{315}{336} = \frac{160+315}{336} = \frac{475}{336}$$

10. *Rearrange suitably and find the sum (i)* $\frac{2}{5}+\frac{5}{6}+\frac{8}{5}+\frac{1}{6}$ *(ii)* $\frac{4}{3}+\frac{1}{4}+\frac{16}{9}+\frac{3}{16}$.

Sol. (*i*) $\frac{2}{5}+\frac{5}{6}+\frac{8}{5}+\frac{1}{6} = \left(\frac{2}{5}+\frac{8}{5}\right)+\left(\frac{5}{6}+\frac{1}{6}\right)$

$$= \left(\frac{2+8}{5}\right)+\left(\frac{5+1}{6}\right) = \frac{10}{5}+\frac{6}{6}$$

$$= \frac{2}{1}+\frac{1}{1} = 2+1 = 3$$

(*ii*) $\frac{4}{3}+\frac{1}{4}+\frac{16}{9}+\frac{3}{16} = \left(\frac{4}{3}+\frac{16}{9}\right)+\left(\frac{1}{4}+\frac{3}{16}\right)$

$$= \left(\frac{4\times3}{3\times3}+\frac{16}{9}\right)+\left(\frac{1\times4}{4\times4}+\frac{3}{16}\right)$$

$$= \left(\frac{12}{9}+\frac{16}{9}\right)+\left(\frac{4}{16}+\frac{3}{16}\right)$$

$$= \left(\frac{12+16}{9}\right)+\left(\frac{4+3}{16}\right)$$

$$= \frac{28}{9}+\frac{7}{16} = \frac{28\times16+7\times9}{144}$$ [∵ LCM of 9 and 16 is 144]

$$= \frac{448+63}{144} = \frac{511}{144}$$

11. *Subtract* $-\frac{8}{9}$ *from* $\frac{11}{24}$. *Also subtract* $\frac{11}{24}$ *from* $-\frac{8}{9}$. *Are the two results the same?*

Sol. On subtracting $\frac{-8}{9}$ from $\frac{11}{24}$, we get

$$\frac{11}{24}-\left(\frac{-8}{9}\right) = \frac{11}{24}+\frac{8}{9} = \frac{(11\times3)+(8\times8)}{72} = \frac{33+64}{72} = \frac{97}{72}$$

On subtracting $\frac{11}{24}$ from $\frac{-8}{9}$, we get

$$\frac{-8}{9}-\frac{11}{24} = \frac{(-8)\times8-11\times3}{72} = \frac{-64-33}{72} = \frac{-97}{72}$$

As $\frac{97}{72} \neq \frac{-97}{72}$, so the two results are not same.

Word Problems

Solved Examples

1. *Huma, Hubna and Seema received a total of ₹ 2,016 as monthly allowance from their mother such that Seema gets $\frac{1}{2}$ of what Huma gets and Hubna gets $1\frac{2}{3}$ times Seema's share. How much money do the three sisters get individually?*

Sol. Let money which Huma gets = ₹ x

$\therefore$ Seema gets money = $\frac{1}{2} \times ₹\, x = ₹\, \frac{x}{2}$ and Hubna gets money = $1\frac{2}{3} \times ₹\, \frac{x}{2} = ₹\left(\frac{5}{3} \times \frac{x}{2}\right) = \frac{₹\, 5x}{6}$

As Huma, Hubna and Seema received a total of ₹ 2,016 as monthly allowance from their mother, so

$$₹\, x + ₹\, \frac{5}{6}x + ₹\, \frac{x}{2} = ₹\, 2{,}016 \quad \Rightarrow x + \frac{5x}{6} + \frac{x}{2} = 2{,}016$$

$$\Rightarrow \quad \frac{6x + 5x + 3x}{6} = 2{,}016 \quad \Rightarrow \frac{14x}{6} = 2{,}016$$

$$\Rightarrow \quad x = \frac{6}{14} \times 2016 = 864$$

$\therefore$ Huma gets = ₹ 864

Hubna gets = $₹\left(\frac{5}{6} \times 864\right) = ₹\, (5 \times 144) = ₹\, 720$

and Seema gets = $₹\left(\frac{1}{2} \times 864\right) = ₹\, 432$

Check: ₹ 864 + ₹ 720 + ₹ 432 = ₹ 2,016

2. *A mother and her two daughters get a room constructed for ₹ 60,000. The elder daughter contributes $\frac{3}{8}$ of her mother's contribution while the younger daughter $\frac{1}{2}$ of her mother's share. How much do the three contribute individually?*

Sol. Let mother's contribution be ₹ x.

$\therefore$ Elder daughter's share = $₹\left(\frac{3}{8} \times x\right) = ₹\, \frac{3x}{8}$

and younger daughter's share = $₹\left(\frac{1}{2} \times x\right) = ₹\, \frac{x}{2}$

As mother and her two daughters got a room constructed for ₹ 60,000

$$\therefore \quad ₹\, x + ₹\, \frac{3}{8}x + ₹\, \frac{x}{2} = ₹\, 60{,}000$$

$$\Rightarrow \quad x + \frac{3x}{8} + \frac{x}{2} = 60{,}000$$

$$\Rightarrow \quad \frac{8x + 3x + 4x}{8} = 60{,}000$$

$$\Rightarrow \quad \frac{15x}{8} = 60{,}000$$

$$\Rightarrow \quad x = 60{,}000 \times \frac{8}{15} = 4{,}000 \times 8 = 32{,}000$$

$\therefore$ Amount contributed by mother = ₹ 32,000

Amount contributed by elder daughter = $₹\left(\frac{3}{8} \times 32{,}000\right) = ₹\, 12{,}000$

and amount contributed by younger daughter = $₹\, \frac{x}{2} = ₹\left(\frac{1}{2} \times 32{,}000\right) = ₹\, 16{,}000$

Check: ₹ 32,000 + ₹ 12,000 + ₹ 16,000 = ₹ 60,000

3. $\frac{1}{6}$ *of the class students are above average,* $\frac{1}{4}$ *are average and rest are below average. If there are 48 students in all, how many students are below average in the class?*

Sol. Total number of students in the class = 48

Number of students who are above average = $\frac{1}{6} \times 48 = 8$

Number of students who are average = $\frac{1}{4} \times 48 = 12$

$\therefore$ Number of students who are below average = $48 - (8 + 12) = 48 - 20 = 28$

Exercise 1.3

There are four options (Q1 to Q6) out of which only one is correct. Choose the correct answer.

1. The additive inverse of $\frac{-5}{13}$ is

(*a*) $\frac{5}{-13}$ (*b*) $\frac{5}{13}$ (*c*) $\frac{13}{5}$ (*d*) $\frac{-13}{7}$

2. $\frac{3}{7} + 0 =$

(*a*) 0 (*b*) $\frac{3}{7}$ (*c*) $\frac{7}{3}$ (*d*) $\frac{-3}{7}$

3. $\frac{1}{4} + \frac{1}{6} =$

(*a*) $\frac{1}{4} - \frac{1}{6}$ (*b*) $\frac{1}{6} - \frac{1}{4}$ (*c*) $\frac{1}{6} + \frac{1}{4}$ (*d*) $-\frac{1}{6} - \frac{1}{4}$

4. $-\left(-\frac{3}{5}\right) =$

(*a*) $\frac{3}{-5}$ (*b*) $\frac{-5}{3}$ (*c*) $\frac{5}{3}$ (*d*) $\frac{3}{5}$

5. Rational numbers are closed under

(*a*) Addition (*b*) Subtraction (*c*) Neither (*a*) nor (*b*) (*d*) Both (*a*) and (*b*)

6. Which of the following expressions shows that rational numbers are associative under addition?

(*a*) $\frac{1}{3} + \left(\frac{1}{5} + \frac{1}{7}\right) = \left(\frac{1}{3} + \frac{1}{5}\right) + \frac{1}{7}$ (*b*) $\frac{1}{3} + \left(\frac{1}{5} + \frac{1}{7}\right) = \frac{1}{3} + \left(\frac{1}{7} + \frac{1}{5}\right)$

(*c*) $\frac{1}{3} + \left(\frac{1}{5} + \frac{1}{7}\right) = \left(\frac{1}{7} + \frac{1}{3}\right) + \frac{1}{5}$ (*d*) $\left(\frac{1}{3} + \frac{1}{5}\right) + \frac{1}{7} = \left(\frac{1}{5} + \frac{1}{3}\right) + \frac{1}{7}$

7. Fill in the blanks:

(*i*) The result of adding two rational numbers is another .... number.

(*ii*) $\frac{a}{b} + \frac{c}{d} =$ ....

(*iii*) Rational numbers have an additive identity of ....

(*iv*) Additive inverse of $\frac{5}{9}$ is ....

(*v*) Rational numbers can be added in any ....

8. State true or false for each of the following statements.

(*i*) $\frac{2}{7} \div 0$ is undefined. (*ii*) $\frac{-3}{4} - \left(\frac{-3}{5}\right) = \frac{-3}{20}$.

(*iii*) The sum of two rational numbers is –7. If one of them is $\frac{-11}{5}$, then other is $\frac{-24}{5}$.

(*iv*) $\frac{5}{7}$ and $\frac{-5}{7}$ are additive inverse of each other.

(*v*) Addition is commutative for rational numbers.

9. Verify that:

(*i*) $\frac{-4}{11}+\frac{-5}{13}=\frac{-5}{13}+\frac{-4}{11}$ (*ii*) $\frac{5}{11}+\frac{-7}{9}=\frac{-7}{9}+\frac{5}{11}$

10. Verify that:

(*i*) $\frac{-1}{3}+\left(\frac{4}{9}+\frac{-8}{13}\right)=\left(\frac{-1}{3}+\frac{4}{9}\right)+\frac{-8}{13}$ (*ii*) $\frac{2}{5}+\left(\frac{-3}{4}+\frac{-4}{7}\right)=\left(\frac{2}{5}+\frac{-3}{4}\right)+\frac{-4}{7}$

11. A city received $\frac{11}{4}$ cm of rainfall on Sunday and $\frac{11}{2}$ cm on Monday. Find the total rainfall in the city on these two days.

12. Geeta spends $\frac{19}{4}$ hours in doing her homework. If she gives $\frac{3}{2}$ hours to Mathematics, find the time she gives to other subjects.

MULTIPLICATION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

We can cancel out common factors from numerator and denominator to find a simpler form of the product.

For example, $$\frac{4}{5}\times\frac{10}{12}=\frac{4\times10}{5\times12}=\frac{1\times2}{1\times3}=\frac{2}{3}$$

$$\frac{4}{5}\times\frac{10}{12}=\frac{4\times10}{5\times12}=\frac{40}{60}$$

You need to simplify, whenever possible

$$\frac{40}{60}=\frac{2\times20}{3\times20}=\frac{2}{3}$$

Properties of Multiplication of Rational Numbers

1. Closure property

The product of two rational numbers is always a rational number.

That is, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

For example, $\frac{1}{3}\times\frac{7}{11}=\frac{1\times7}{3\times11}=\frac{7}{33}$, which is a rational number.

2. Commutative property

Multiplication is commutative for rational numbers.

That is, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$$

Clearly, two rational numbers can be multiplied in any order.

For example, $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}=\frac{4}{5}\times\frac{2}{3}$.

3. Associative property

Multiplication is associative for rational numbers.

That is, for any three rational numbers, $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$$

Clearly, while multiplying three rational numbers, they can be grouped in any order.

For example, $\frac{4}{7}\times\left(\frac{2}{3}\times\frac{5}{6}\right)=\frac{4}{7}\times\frac{10}{18}=\frac{40}{126}$ and $\left(\frac{4}{7}\times\frac{2}{3}\right)\times\frac{5}{6}=\frac{8}{21}\times\frac{5}{6}=\frac{40}{126}$

Thus, $\frac{4}{7}\times\left(\frac{2}{3}\times\frac{5}{6}\right)=\left(\frac{4}{7}\times\frac{2}{3}\right)\times\frac{5}{6}$

4. Existence of multiplicative identity

1 is called the multiplicative identity for rational numbers because the product of any rational number and 1 is the rational number itself.

That is, for any rational number $\frac{a}{b}$, we have

$$\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$$

For example, $\frac{5}{7}\times 1=\frac{5}{7}\times\frac{1}{1}=\frac{5\times 1}{7\times 1}=\frac{5}{7}$ and $1\times\frac{5}{7}=\frac{1}{1}\times\frac{5}{7}=\frac{1\times 5}{1\times 7}=\frac{5}{7}$

Therefore, $\frac{5}{7}\times 1=1\times\frac{5}{7}=\frac{5}{7}$

5. Existence of multiplicative inverse

Every non-zero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

So, $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$

$\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

For example, $\frac{7}{15}\times\frac{15}{7}=\frac{7\times 15}{15\times 7}=\frac{105}{105}=1$

Thus, $\frac{7}{15}$ and $\frac{15}{7}$ are multiplicative inverse of each other.

Remember

Every number has a reciprocal except 0 ($\frac{1}{0}$ is undefined). Reciprocal of 1 is 1 and reciprocal of (–1) is –1.

For example,

(*i*) Multiplicative inverse or reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, since $\frac{3}{4}\times\frac{4}{3}=\frac{4}{3}\times\frac{3}{4}=1$

(*ii*) Reciprocal of $\frac{-5}{9}$ is $\frac{-9}{5}$, since $\frac{-5}{9}\times\frac{-9}{5}=\frac{-9}{5}\times\frac{-5}{9}=1$

(*iii*) Reciprocal of –7 is $\frac{-1}{7}$, since $-7\times\frac{-1}{7}=\frac{-7}{1}\times\frac{(-1)}{7}=\frac{\{(-7)\times(-1)\}}{1\times 7}=\frac{7}{7}=1$

6. Distributive property of multiplication over addition

Multiplication of rational numbers is distributive over addition.

That is, for any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$$

For example, for three rational numbers $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$, we have

$$\frac{2}{3}\times\left(\frac{3}{4}+\frac{5}{6}\right)=\frac{2}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{6}$$

Iere $\qquad$ $\text{L.H.S.} = \frac{2}{3}\times\left(\frac{3}{4}+\frac{5}{6}\right) = \frac{2}{3}\times\left(\frac{9+10}{12}\right) = \frac{2}{3}\times\frac{19}{12} = \frac{19}{18}$

nd $\qquad$ $\text{R.H.S.} = \frac{2}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{6} = \frac{6}{12}+\frac{10}{18} = \frac{1}{2}+\frac{5}{9} = \frac{9+10}{18} = \frac{19}{18}$

'hus, $\qquad$ L.H.S. = R.H.S.

. Distributive property of multiplication over subtraction

Iultiplication of rational numbers is distributive over subtraction.

'hat is, for three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$, we have

$$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right) = \frac{a}{b}\times\frac{c}{d}-\frac{a}{b}\times\frac{e}{f}$$

or example, for rational numbers $\frac{3}{5}$, $\frac{1}{7}$ and $\frac{4}{9}$, we have

$$\frac{3}{5}\times\left(\frac{1}{7}-\frac{4}{9}\right) = \frac{3}{5}\times\frac{1}{7}-\frac{3}{5}\times\frac{4}{9}$$

Iere, $\qquad$ $\text{L.H.S.} = \frac{3}{5}\times\left(\frac{1}{7}-\frac{4}{9}\right) = \frac{3}{5}\times\left(\frac{9-28}{63}\right) = \frac{3}{5}\times\frac{-19}{63} = \frac{-19}{5\times 21} = \frac{-19}{105}$

nd $\qquad$ $\text{R.H.S.} = \frac{3}{5}\times\frac{1}{7}-\frac{3}{5}\times\frac{4}{9} = \frac{3}{35}-\frac{12}{45} = \frac{(3\times 9)-(12\times 7)}{315}$

[∵ LCM of 35 and 45 is 315]

$$= \frac{27-84}{315} = \frac{-57}{315} = \frac{-19}{105}$$

hus, $\qquad$ L.H.S = R.H.S

Solved Examples

1. *Multiply:*

(*i*) $\frac{3}{7}$ *by* $\frac{14}{9}$ $\qquad$ (*ii*) $\frac{5}{-13}$ *by* $\frac{-26}{15}$ $\qquad$ (*iii*) $\frac{-8}{25}$ *by* $\frac{-5}{16}$

ol. (*i*) $\frac{3}{7}\times\frac{14}{9} = \frac{1\times 2}{1\times 3} = \frac{2}{3}$ OR $\frac{3}{7}\times\frac{14}{9} = \frac{3\times 14}{7\times 9} = \frac{42}{63} = \frac{42\div 21}{63\div 21} = \frac{2}{3}$

(*ii*) $\frac{5}{-13}\times\frac{-26}{15} = \frac{5\times(-26)}{-13\times 15} = \frac{1\times 2}{1\times 3} = \frac{2}{3}$

(*iii*) $\frac{-8}{25}\times\frac{-5}{16} = \frac{-8\times(-5)}{25\times 16} = \frac{-1\times(-1)}{5\times 2} = \frac{1}{10}$

2. *Multiply* $\frac{-6}{8}$ *by* $\frac{14}{21}$.

ol. $\frac{-6}{8}\times\frac{14}{21} = \frac{-3}{4}\times\frac{2}{3} = \frac{-1\times 1}{2\times 1} = \frac{-1}{2}$ OR $\frac{-6}{8}\times\frac{14}{21} = \frac{-6\times 14}{8\times 21} = \frac{-84}{168} = \frac{-84\div 84}{168\div 84} = \frac{-1}{2}$

3. *Multiply* $-2\frac{1}{6}$ *by* $1\frac{1}{5}$.

ol. $-2\frac{1}{6}\times 1\frac{1}{5} = \frac{-13}{6}\times\frac{6}{5} = \frac{-13\times 1}{1\times 5} = \frac{-13}{5}$

4. *Simplify* $\frac{1}{-3}\times\frac{3}{7}\times\frac{7}{9}$.

Sol. $\frac{1}{-3}\times\frac{3}{7}\times\frac{7}{9} = \frac{1\times1\times1}{-1\times1\times9} = \frac{1}{-9} = -\frac{1}{9}$

5. *Verify that* $\frac{3}{7}\times\frac{-17}{8} = \frac{-17}{8}\times\frac{3}{7}$.

Sol.

$$\text{L.H.S.} = \frac{3}{7}\times\frac{-17}{8} = \frac{3\times(-17)}{7\times8} = \frac{-51}{56}$$

$$\text{R.H.S.} = \frac{-17}{8}\times\frac{3}{7} = \frac{-17\times3}{8\times7} = \frac{-51}{56}$$

$\therefore$ L.H.S. = R.H.S. is verified.

6. *Find the product (i)* $\frac{-3}{5}\times\frac{35}{27}$ *(ii)* $\frac{4}{11}\times\left(\frac{-1}{6}\right)$

Sol. (*i*) $\frac{-3}{5}\times\frac{35}{27} = \frac{-3\times35}{5\times27} = \frac{-105}{135} = \frac{-7\times15}{9\times15} = \frac{-7}{9}$

(*ii*) $\frac{4}{11}\times\left(\frac{-1}{6}\right) = \frac{4\times(-1)}{11\times6} = \frac{-4}{66} = \frac{-2\times2}{2\times33} = \frac{-2}{33}$

7. *Evaluate* $2\frac{3}{4}+2\frac{3}{4}+...$ *thirty times.*

Sol. $2\frac{3}{4}+2\frac{3}{4}+...$ thirty times $= 2\frac{3}{4}\times30$

$$= \left(\frac{2\times4+3}{4}\right)\times30 = \frac{11}{4}\times\frac{30}{1} = \frac{11\times30}{4\times1}$$

$$= \frac{330}{4} = \frac{165\times2}{2\times2} = \frac{165}{2} = 82\frac{1}{2}$$

8. *Find the multiplicative inverse of the following:*

(*i*) -13 (*ii*) $\frac{-13}{19}$ (*iii*) $\frac{1}{5}$ (*iv*) $\frac{-5}{8}\times\frac{-3}{7}$

(*v*) $-1\times\frac{-2}{5}$ (*vi*) -1

Sol. We know that a rational number $\frac{c}{d}$ is the multiplicative inverse or reciprocal of another rational numb of $\frac{a}{b}$ if $\frac{a}{b}\times\frac{c}{d} = 1$

(*i*) The multiplicative inverse of -13 is $\frac{-1}{13}$ because $\frac{-1}{13}\times(-13) = 1$.

(*ii*) The multiplicative inverse of $\frac{-13}{19}$ is $\frac{-19}{13}$ because $\frac{-13}{19}\times\frac{-19}{13} = 1$.

(*iii*) The multiplicative inverse of $\frac{1}{5}$ is 5 because $\frac{1}{5}\times5 = 1$

(*iv*) We have $\frac{-5}{8}\times\frac{-3}{7} = \frac{(-5)\times(-3)}{8\times7} = \frac{15}{56}$

Multiplicative inverse of $\frac{15}{56}$ is $\frac{56}{15}$ because $\frac{15}{56}\times\frac{56}{15} = 1$

(*v*) We have $-1\times\frac{-2}{5} = \frac{(-1)\times(-2)}{5} = \frac{2}{5}$

Multiplicative inverse of $\frac{2}{5}$ is $\frac{5}{2}$ because $\frac{2}{5}\times\frac{5}{2} = 1$

(*vi*) Multiplicative inverse of -1 is -1 because $-1\times(-1) = 1$

9. *Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?*

Sol. We have $-1\frac{1}{8} = -\left(\frac{8+1}{8}\right) = -\frac{9}{8}$

If $\frac{8}{9}$ is the multiplicative inverse of $-\frac{9}{8}$, then we must have $-\frac{9}{8}\times\frac{8}{9}$ equal to 1 but $-\frac{9}{8}\times\frac{8}{9} = -1$.

Hence, $\frac{8}{9}$ is not the multiplicative inverse of $-\frac{9}{8}$.

10. *Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?*

Sol. We know that $0.3 = \frac{3}{10}$ and $3\frac{1}{3} = \frac{10}{3}$

We observe that $\frac{10}{3}\times\frac{3}{10} = 1$

Hence, 0.3 is the multiplicative inverse of $3\frac{1}{3}$.

11. *Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.*

Sol. For any rational number $\frac{p}{q}$, we have another non-zero rational number $\frac{q}{p}$, which is called the reciprocal of $\frac{p}{q}$.

So, the reciprocal of $\frac{-7}{16}$ is $\frac{16}{-7}$.

Now, $\frac{6}{13}\times\frac{16}{-7} = -\frac{6\times16}{13\times7} = -\frac{96}{91}$

12. *Write:*

(*i*) *The rational number that does not have a reciprocal.*

(*ii*) *The rational numbers that are equal to their reciprocals.*

(*iii*) *The rational number that is equal to its negative.*

Sol. (*i*) As there is no rational number which when multiplied by 0 gives 1, so zero (0) as a rational number has no reciprocal.

(*ii*) 1 is a rational number which is equal to its reciprocal because, $1 \times 1 = 1$

Again, –1 is a rational number which is equal to its reciprocal because, $-1 \times (-1) = 1$.

Hence, 1 and –1 are rational numbers that are equal to their reciprocals.

(*iii*) Negative of a rational number a is $-a$ and the negative of $-a$ is a.

We have $a + (-a) = (-a) + a = 0$

As $0 + 0 = 0$, so the negative of rational number 0 is 0.

13. *Name the property under multiplication used in each of the following.*

(*i*) $\frac{-4}{5}\times1 = 1\times\frac{-4}{5} = -\frac{4}{5}$ (*ii*) $-\frac{13}{17}\times\frac{-2}{7} = \frac{-2}{7}\times\frac{-13}{17}$ (*iii*) $\frac{-19}{29}\times\frac{29}{-19} = 1$

Sol. (*i*) 1 is the multiplicative identity for rational numbers. If we multiply any rational number by 1, we get the same rational number.

(*ii*) Commutative property of multiplication.

(*iii*) Multiplicative inverse.

14. *Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ as $\left(\frac{1}{3}\times6\right)\times\frac{4}{3}$.*

Sol. Associativity for multiplication.

15. *Using associative property for multiplication of rational numbers, find the product* $\frac{1}{2}\times\frac{3}{4}\times\frac{4}{5}$ *in two ways.*

Sol. $\left(\frac{1}{2}\times\frac{3}{4}\right)\times\frac{4}{5} = \frac{3}{8}\times\frac{4}{5} = \frac{3}{10}$

and $\frac{1}{2}\times\left(\frac{3}{4}\times\frac{4}{5}\right) = \frac{1}{2}\times\frac{3}{5} = \frac{3}{10}$

Hence, $\left(\frac{1}{2}\times\frac{3}{4}\right)\times\frac{4}{5} = \frac{1}{2}\times\left(\frac{3}{4}\times\frac{4}{5}\right)$

16. *Using appropriate properties find:*

(*i*) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$ (*ii*) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

Sol. (*i*) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6} = -\frac{2}{3}\times\frac{3}{5}-\frac{3}{5}\times\frac{1}{6}+\frac{5}{2}$ [Rearranging the terms]

$= -\frac{3}{5}\times\frac{2}{3}-\frac{3}{5}\times\frac{1}{6}+\frac{5}{2}$ [Commutative property of multiplication]

$= -\frac{3}{5}\left(\frac{2}{3}+\frac{1}{6}\right)+\frac{5}{2}$ [Distributive property]

$= -\frac{3}{5}\left(\frac{4+1}{6}\right)+\frac{5}{2}$

$= -\frac{3}{5}\times\frac{5}{6}+\frac{5}{2} = \frac{-1}{2}+\frac{5}{2}$

$= \frac{-1+5}{2} = \frac{4}{2} = 2$

(*ii*) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5} = \frac{2}{5}\times\left(-\frac{3}{7}\right)+\frac{1}{14}\times\frac{2}{5}-\frac{1}{6}\times\frac{3}{2}$ [Rearranging the terms]

$= \frac{2}{5}\times\left(-\frac{3}{7}\right)+\frac{2}{5}\times\frac{1}{14}-\frac{1}{6}\times\frac{3}{2}$ [Commutative property of multiplication]

$= \frac{2}{5}\times\left[\left(-\frac{3}{7}\right)+\frac{1}{14}\right]-\frac{1}{2}\times\frac{1}{2}$ [Distributive property]

$= \frac{2}{5}\times\left(\frac{-6+1}{14}\right)-\frac{1}{4}$

$= \frac{2}{5}\times\frac{-5}{14}-\frac{1}{4}$

$= -\frac{1}{7}-\frac{1}{4} = \frac{-4-7}{28} = \frac{-11}{28}$

Miscellaneous Solved Examples

There are four options (Q1 to Q6) out of which only one is correct. Choose the correct answer.

1. *Which of the following expressions shows that rational numbers are associative under multiplication?*

(*a*) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{3}{5}$ (*b*) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right)=\frac{2}{3}\times\left(\frac{3}{5}\times\frac{-6}{7}\right)$

(*c*) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right)=\left(\frac{3}{5}\times\frac{2}{3}\right)\times\frac{-6}{7}$ (*d*) $\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{3}{5}=\left(\frac{-6}{7}\times\frac{2}{3}\right)\times\frac{3}{5}$

Sol. Expression in (*a*), $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{3}{5}$ shows that rational numbers are associative under multiplication.

Hence, (*a*) is the correct answer.

2. *Multiplicative inverse of a negative rational number is*

(*a*) *a positive rational number* (*b*) *a negative rational number*

(*c*) 0 (*d*) 1

Sol. Consider –3. Its multiplicative inverse is $\frac{1}{-3}$.

So, multiplicative inverse of a negative rational number is a negative rational number.

Hence, (*b*) is the correct answer.

3. *To get the product 1, we should multiply* $\frac{8}{21}$ *by*

(*a*) $\frac{8}{21}$ (*b*) $\frac{-8}{21}$ (*c*) $\frac{21}{8}$ (*d*) $\frac{-21}{8}$

Sol. To get the product 1, we should multiply $\frac{8}{21}$ by its multiplicative inverse *i.e.*, $\frac{21}{8}$.

as $\frac{8}{21}\times\frac{21}{8}=1$

Hence, (*c*) is the correct answer.

4. *The multiplicative inverse of* $-1\frac{1}{7}$ *is*

(*a*) $\frac{8}{7}$ (*b*) $\frac{-8}{7}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{7}{-8}$

Sol. We have $-1\frac{1}{7}=\frac{-8}{7}$

Its multiplicative inverse is $\frac{7}{-8}$ because $\left(\frac{-8}{7}\times\frac{7}{-8}\right)=1$

Hence, (*d*) is the correct answer.

5. *The reciprocal of* 0 *is*

(*a*) 1 (*b*) –1 (*c*) 0 (*d*) Not defined

Sol. The reciprocal of 0 is $\frac{1}{0}$. As division by zero is undefined, so $\frac{1}{0}$ is undefined.

Hence, (*d*) is the correct answer.

6. *If y be the reciprocal of rational number x, then the reciprocal of y will be*

(*a*) *x* (*b*) *y* (*c*) $\frac{x}{y}$ (*d*) $\frac{y}{x}$

Sol. As y is the reciprocal of x, so $x \times y = 1$, *i.e.*, $xy = 1$

Now, the product of xy is 1. So, the reciprocal of y is x.

Hence, (*a*) is the correct answer.

7. *Fill in the blanks to make the statements true.*

(*i*) *The reciprocal of a positive rational number is* ....

(*ii*) *The reciprocal of a negative rational number is* ....

(*iii*) *Zero has* .... *reciprocal.*

(*iv*) *The numbers* .... *and* .... *are their own reciprocal.*

(*v*) *The reciprocal of* $\frac{2}{5}\times\left(\frac{-4}{9}\right)$ *is* ....

(*vi*) *For rational numbers* $\frac{a}{b}, \frac{c}{d}$ *and* $\frac{e}{f}$ *we have* $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) =$ .... + ...

(*vii*) $\frac{1}{5} \times \left[\frac{2}{7} + \frac{3}{8}\right] = \left[\frac{1}{5} \times \frac{2}{7}\right] +$ ....

Sol. (*i*) The reciprocal of a positive rational number is positive rational number.

(*ii*) The reciprocal of a negative rational number is negative rational number.

(*iii*) Zero has no reciprocal.

(*iv*) The numbers 1 and –1 are their own reciprocal.

(*v*) The reciprocal of $\frac{2}{5} \times \left(\frac{-4}{9}\right)$ is $\frac{-45}{8}$.

$\left[\because \frac{2}{5} \times \left(\frac{-4}{9}\right) = \frac{2 \times (-4)}{5 \times 9} = \frac{-8}{45} \therefore \text{Reciprocal of } \frac{-8}{45} \text{ is } \frac{-45}{8} \text{ because } \frac{-8}{45} \times \frac{-45}{8} = 1\right]$

(*vi*) For rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ we have $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

(*vii*) $\frac{1}{5} \times \left[\frac{2}{7} + \frac{3}{8}\right] = \frac{1}{5} \times \frac{2}{7} + \frac{1}{5} \times \frac{3}{8}$.

8. *In each of the following, state whether the statements are true (T) or false (F).*

(*i*) 1 *is the only number which is its own reciprocal.*

(*ii*) *For all rational numbers x and y,* $x \times y = y \times x$

(*iii*) *For rational numbers x, y and z,* $x + (y \times z) = (x + y) \times (x + z)$

(*iv*) *For all rational numbers a, b and c,* $a(b + c) = ab + ac$

(*v*) –1 *is not the reciprocal of any rational number.*

Sol. (*i*) It is a false statement. –1 is another rational which is its own reciprocal.

(*ii*) The given statement is true because multiplication is commutative for rational numbers.

(*iii*) The given statement is false because addition is not distributive over multiplication for rational numbers.

(*iv*) The given statement is true because multiplication is distributive over addition for rational numbers.

(*v*) The given statement is false because –1 is the reciprocal of –1.

9. *Tell which property allows you to compute* $\frac{1}{5} \times \left[\frac{5}{6} \times \frac{7}{9}\right]$ *as* $\left[\frac{1}{5} \times \frac{5}{6}\right] \times \frac{7}{9}$

Sol. $\frac{1}{5} \times \left[\frac{5}{6} \times \frac{7}{9}\right]$ can be computed as $\left[\frac{1}{5} \times \frac{5}{6}\right] \times \frac{7}{9}$ by associative property of multiplication.

10. *Verify the property* $x \times y = y \times x$ *of rational numbers by using:*

(*i*) $x = \frac{-5}{7}$ *and* $y = \frac{14}{15}$ (*ii*) $x = \frac{-3}{8}$ *and* $y = \frac{-4}{9}$

Sol. (*i*) Here, $x = \frac{-5}{7}$ and $y = \frac{14}{15}$

Since, $x \times y = \frac{-5}{7} \times \frac{14}{15} = \frac{-1 \times 2}{1 \times 3} = \frac{-2}{3}$

Also, $y \times x = \frac{14}{15} \times \frac{-5}{7} = \frac{2 \times (-1)}{3 \times 1} = \frac{-2}{3}$

Hence, $x \times y = y \times x$ is verified.

(*ii*) Here, $x = \frac{-3}{8}$ and $y = \frac{-4}{9}$

Since, $x \times y = \frac{-3}{8} \times \frac{-4}{9} = \frac{(-1) \times (-1)}{2 \times 3} = \frac{1}{6}$

Also, $y \times x = \frac{-4}{9} \times \frac{-3}{8} = \frac{(-1) \times (-1)}{3 \times 2} = \frac{1}{6}$

Hence, $x \times y = y \times x$ is verified.

11. *Verify the property $x \times (y \times z) = (x \times y) \times z$ of rational numbers by using*

(*i*) $x = 1, y = \frac{-1}{2}$ *and* $z = \frac{1}{4}$ (*ii*) $x = \frac{2}{3}, y = \frac{-3}{7}$ *and* $z = \frac{1}{2}$

What is the name of this property?

Sol. (*i*) $x \times (y \times z) = 1 \times \left(\frac{-1}{2} \times \frac{1}{4}\right) = 1 \times \left(\frac{-1 \times 1}{2 \times 4}\right) = 1 \times \frac{-1}{8} = \frac{-1}{8}$

Also, $(x \times y) \times z = \left(1 \times \frac{-1}{2}\right) \times \frac{1}{4} = \frac{1 \times (-1)}{2} \times \frac{1}{4} = \frac{-1}{2} \times \frac{1}{4} = \frac{(-1) \times 1}{2 \times 4} = \frac{-1}{8}$

Hence, $x \times (y \times z) = (x \times y) \times z$ is verified.

(*ii*) $x \times (y \times z) = \frac{2}{3} \times \left(\frac{-3}{7} \times \frac{1}{2}\right) = \frac{2}{3} \times \frac{(-3) \times 1}{7 \times 2} = \frac{2}{3} \times \frac{-3}{14} = \frac{1 \times (-1)}{1 \times 7} = \frac{-1}{7}$

Also, $(x \times y) \times z = \left(\frac{2}{3} \times \frac{-3}{7}\right) \times \frac{1}{2} = \frac{2 \times (-3)}{3 \times 7} \times \frac{1}{2}$

$= \frac{2 \times (-1)}{1 \times 7} \times \frac{1}{2} = \frac{-2}{7} \times \frac{1}{2} = \frac{(-1) \times 1}{7 \times 1} = \frac{-1}{7}$

Hence, $x \times (y \times z) = (x \times y) \times z$ is verified. The name of this property is Associative Property of Multiplication.

12. *Verify the property $x \times (y + z) = x \times y + x \times z$ of rational numbers by using $x = \frac{-1}{2}, y = \frac{2}{3}, z = \frac{3}{4}$*

Sol. L.H.S. $= x \times (y + z) = \frac{-1}{2} \times \left(\frac{2}{3} + \frac{3}{4}\right) = \frac{-1}{2}\left(\frac{2 \times 4 + 3 \times 3}{12}\right)$

$= \frac{-1}{2} \times \frac{17}{12} = \frac{-1 \times 17}{2 \times 12} = \frac{-17}{24}$

R.H.S. $= x \times y + x \times z = \left(\frac{-1}{2}\right) \times \frac{2}{3} + \left(\frac{-1}{2}\right) \times \frac{3}{4} = \frac{(-1) \times 2}{2 \times 3} + \frac{(-1) \times 3}{2 \times 4} = \frac{-2}{6} + \frac{-3}{8}$

$= \frac{(-2) \times 4 + (-3) \times 3}{24} = \frac{-8 + (-9)}{24} = \frac{-8 - 9}{24} = \frac{-17}{24}$

Since L.H.S. = R.H.S., so the property of $x \times (y + z) = x \times y + x \times z$ is verified.

13. *Use the distributivity of multiplication of rational numbers over addition to simplify*

(*i*) $\frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1}\right]$ (*ii*) $\frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15}\right]$ (*iii*) $\frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4}\right]$ (*iv*) $\frac{3}{4} \times \left[\frac{8}{9} - 40\right]$

Sol. We know that for rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$,

$$\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$$

[Multiplication distributes over addition for rational numbers]

(*i*) $\frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1}\right] = \frac{3}{5} \times \frac{35}{24} + \frac{3}{5} \times \frac{10}{1} = \frac{1 \times 7}{1 \times 8} + \frac{3 \times 2}{1 \times 1} = \frac{7}{8} + \frac{6}{1}$

$= \frac{7 + (6 \times 8)}{8} = \frac{7 + 48}{8} = \frac{55}{8}$

(*ii*) $\frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15}\right] = \frac{-5}{4} \times \frac{8}{5} + \frac{-5}{4} \times \frac{16}{15} = (-1) \times 2 + \frac{(-1)(4)}{1 \times 3} = -2 - \frac{4}{3}$

$= \frac{(-2)(3) - 4}{3} = \frac{-6 - 4}{3} = \frac{-10}{3}$

(iii) $\frac{2}{7}\times\left[\frac{7}{16}-\frac{21}{4}\right]=\frac{2}{7}\times\frac{7}{16}-\frac{2}{7}\times\frac{21}{4}=\frac{1\times1}{1\times8}-\frac{1\times3}{1\times2}=\frac{1}{8}-\frac{3}{2}=\frac{1-(3\times4)}{8}$

$=\frac{1-12}{8}=\frac{-11}{8}$

(iv) $\frac{3}{4}\times\left[\frac{8}{9}-40\right]=\frac{3}{4}\times\frac{8}{9}-\frac{3}{4}\times\frac{40}{1}=\frac{1\times2}{1\times3}-\frac{3\times10}{1\times1}=\frac{2}{3}-\frac{30}{1}$

$=\frac{(2\times1)-(30\times3)}{3}=\frac{2-90}{3}=\frac{-88}{3}$

Word Problems

Solved Examples

1. *Find the product of additive inverse and multiplicative inverse of $-\frac{1}{3}$.*

Sol. Additive inverse of $-\frac{1}{3}$ is $\frac{1}{3}$ and multiplicative inverse of $-\frac{1}{3}$ is $-\frac{3}{1}$

Their product $=\frac{1}{3}\times\left(-\frac{3}{1}\right)=\frac{1\times(-3)}{3\times1}=\frac{-3}{3}=\frac{-1}{1}=-1$

2. *The cost of one litre of petrol is ₹ $62\frac{1}{2}$. Find the cost of 12 litres of petrol.*

Sol. Cost of one litre of petrol is ₹ $62\frac{1}{2}$ = ₹ $\frac{125}{2}$

∴ Cost of 12 litres of petrol = ₹ $\frac{125}{2}\times\frac{12}{1}$ = ₹ $\left(\frac{125\times6}{1\times1}\right)$ = ₹ $\left(\frac{750}{1}\right)$ = ₹ 750

3. *The average speed of a bus is $45\frac{1}{3}$ km/hr. Find the distance covered by the bus in $1\frac{3}{4}$ hours.*

Sol. Average speed of bus = $45\frac{1}{3}$ km/hr = $\frac{136}{3}$ km/hr

∵ Distance covered by the bus in 1 hour = $\frac{136}{3}$ km

∴ Distance covered by the bus in $1\frac{3}{4}$ hours $\left(=\frac{7}{4}\text{ hours}\right)=\left(\frac{136}{3}\times\frac{7}{4}\right)$ km

$=\frac{34\times7}{3\times1}$ km $=\frac{238}{3}$ km $=79\frac{1}{3}$ km

Exercise 1.4

There are four options (Q1 to Q5) out of which only one is correct. Choose the correct answer.

1. The multiplicative inverse of $-1\frac{1}{3}$ =

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{-3}{4}$

2. The reciprocal of $\frac{-2}{5}\times\left(\frac{-8}{11}\right)$ =

(a) $\frac{-55}{16}$ (b) $\frac{55}{16}$ (c) $\frac{11}{55}$ (d) $\frac{-16}{55}$

3. For any rational number $\frac{5}{9}$, we have $\frac{5}{9}\times1$ =

(a) $\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{-5}{9}$ (d) $\frac{9}{-5}$

4. For any rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=$

(a) $\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ (b) $\left(\frac{c}{d}\times\frac{a}{b}\right)\times\frac{e}{f}$ (c) $\left(\frac{a}{b}\times\frac{e}{f}\right)\times\frac{c}{d}$ (d) None of these

5. $\left(\frac{-2}{5}\right)\times\left(\frac{-7}{9}\right)=$

(a) $\left(\frac{2}{-5}\right)\times\left(\frac{-7}{9}\right)$ (b) $\left(\frac{-7}{9}\right)\times\left(\frac{-2}{5}\right)$ (c) $\left(\frac{2}{-5}\right)\times\left(\frac{9}{-7}\right)$ (d) $\left(\frac{-2}{5}\right)\times\left(\frac{9}{-7}\right)$

6. Fill in the blanks:

(i) Two rational numbers can be multiplied in any ....

(ii) 1 is called the .... for rational numbers.

(iii) $\frac{b}{a}$ is called the .... of $\frac{a}{b}$.

(iv) 0 has no ....

7. For rational numbers x and y, verify the property $x \times y = y \times x$ by taking

(i) $x=\frac{-3}{5}, y=\frac{10}{7}$ (ii) $x=\frac{2}{7}, y=\frac{-11}{76}$

8. For rational numbers x, y and z, verify the property $x \times (y \times z) = (x \times y) \times z$ if

(i) $x=\frac{5}{4}, y=\frac{-11}{3}, z=\frac{2}{7}$ (ii) $x=\frac{1}{2}, y=\frac{5}{3}, z=\frac{-4}{5}$

9. Use the distributivity of multiplication over addition and simplify.

(i) $\frac{5}{3}\times\left(\frac{24}{5}+\frac{9}{10}\right)$ (ii) $\frac{2}{3}\times\left(\frac{9}{5}+\frac{6}{7}\right)$ (iii) $\frac{-3}{4}\times\left(\frac{5}{2}+\frac{7}{6}\right)$

10. State the property used in each of the following:

(i) $\frac{2}{5}\times\frac{3}{7}=\frac{3}{7}\times\frac{2}{5}$ (ii) $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$

(iii) $\frac{3}{4}\times\left(\frac{2}{5}+\frac{1}{7}\right)=\frac{3}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{1}{7}$

(iv) For any rational number $\frac{a}{b}$, $\left(\frac{a}{b}\times 0\right)=\left(0\times\frac{a}{b}\right)=0$

(v) $\frac{-4}{7}\times 1 = 1\times\frac{-4}{7}=\frac{-4}{7}$

11. Rajni bought $2\frac{1}{2}$ kg onion at ₹ $30\frac{1}{2}$ per kg. Find the amount spent by Rajni.

12. The cost of 1 m ribbon is ₹ 75. Find the cost of $\frac{7}{5}$ metres of ribbon.

DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, such that $\frac{c}{d}\neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{1}{\frac{c}{d}}=\frac{a}{b}\times\frac{d}{c} \qquad \left[\text{Reciprocal of } \frac{c}{d} \text{ is } \frac{d}{c}\right]$$

Hence, division is the inverse process of multiplication.

Dividend: The number to be divided is called the dividend.

Divisor: The number which divides the dividend is called the divisor.

Quotient: When dividend is divided by divisor, the result of the division is called the quotient.

If $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is the dividend, $\frac{c}{d}$ is the divisor and $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ is the quotient.

Note: It should be noted that division by 0 is not defined.

Example 1. *Divide* $\frac{5}{14}$ *by* $\frac{3}{7}$.

Sol. $\frac{5}{14} \div \frac{3}{7} = \frac{5}{14} \times \frac{7}{3} = \frac{5 \times 1}{2 \times 3} = \frac{5}{6}$ $\left[\because \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}\right]$

Example 2. *Divide* $\frac{9}{22}$ *by* $\frac{5}{11}$.

Sol. $\frac{9}{22} \div \frac{5}{11} = \frac{9}{22} \times \frac{11}{5} = \frac{(9 \times 11)}{(22 \times 5)} = \frac{9 \times 1}{2 \times 5} = \frac{9}{10}$

Example 3. *Divide* $-\frac{9}{25}$ *by* $\frac{(-3)}{5}$.

Sol. $\frac{-9}{25} \div \frac{(-3)}{5} = \frac{(-9)}{25} \times \frac{5}{(-3)} = \frac{3 \times 1}{5 \times 1} = \frac{3}{5}$

Properties of Division of Rational Numbers

1. Closure property

Rational numbers (non-zero) are closed under division.

That is, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers such that $\frac{c}{d} \neq 0$, then $\left(\frac{a}{b} \div \frac{c}{d}\right)$ is a rational number.

For example, $\left(\frac{5}{9} \div \frac{-5}{9}\right) = \frac{5}{9} \times \frac{9}{-5} = \frac{5 \times 9}{9 \times (-5)} = \frac{45}{-45} = \frac{1}{-1} = -1$, which is a rational number.

2. Property of 1 for division of rational numbers

(*i*) *Any rational number divided by* 1 *gives the same rational number.*

That is, for every rational number $\frac{a}{b}$, we have $\frac{a}{b} \div 1 = \frac{a}{b}$.

For example, $\frac{7}{13} \div 1 = \frac{7}{13}$ and $\frac{9}{-2} \div 1 = \frac{9}{-2}$

(*ii*) *Any non-zero rational number divided by itself gives* 1 *as the quotient.*

That is, for every non-zero rational number $\frac{a}{b}$, we have $\frac{a}{b} \div \frac{a}{b} = 1$

For example, $\frac{3}{7} \div \frac{3}{7} = \frac{3}{7} \times \frac{7}{3} = \frac{(3 \times 7)}{(7 \times 3)} = \frac{21}{21} = 1$

3. Commutative Property

Division is not commutative for rational numbers:

For example, $\frac{-5}{3} \div \frac{2}{7} = \frac{-5}{3} \times \frac{7}{2} = \frac{-5 \times 7}{3 \times 2} = \frac{-35}{6}$ and $\frac{2}{7} \div \left(\frac{-5}{3}\right) = \frac{2}{7} \times \frac{3}{-5} = \frac{2 \times 3}{7 \times (-5)} = \frac{6}{-35}$

As $\frac{-35}{6}$ is not the same as $\frac{6}{-35}$

So, $\frac{-5}{3} \div \frac{2}{7} \neq \frac{2}{7} \div \left(\frac{-5}{3}\right)$

4. Associative Property

Division is not associative for rational numbers.

For example, let us check if $\frac{1}{3} \div \left(\frac{-1}{2} \div \frac{3}{4}\right) = \left[\frac{1}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{3}{4}$

$$\text{L.H.S.} = \frac{1}{3} \div \left(\frac{-1}{2} \div \frac{3}{4}\right) = \frac{1}{3} \div \left(\frac{-1}{2} \times \frac{4}{3}\right) \qquad \left[\text{Reciprocal of } \frac{3}{4} \text{ is } \frac{4}{3}\right]$$

$$= \frac{1}{3} \div \left(\frac{-4}{6}\right) = \frac{1}{3} \times \frac{6}{-4} = \frac{1 \times 6}{3 \times (-4)} = \frac{6}{-12} = -\frac{1}{2}$$

$$\text{R.H.S.} = \left[\frac{1}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{3}{4} = \left(\frac{1}{3} \times \frac{2}{-1}\right) \div \frac{3}{4} = \frac{2}{-3} \div \frac{3}{4} = \frac{2}{-3} \times \frac{4}{3}$$

$$= \frac{2 \times 4}{-3 \times 3} = \frac{8}{-9} = \frac{-8}{9}$$

As L.H.S. is not the same as R.H.S., so division is not associative for rational numbers.

Solved Examples

1. *Find*: (*i*) $\frac{5}{3} \div \frac{25}{18}$ (*ii*) $\frac{-6}{25} \div \frac{3}{5}$ (*iii*) $\frac{11}{24} \div \frac{(-5)}{8}$

Sol. We know that, if $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

(*i*) $\frac{5}{3} \div \frac{25}{18} = \frac{5}{3} \times \frac{18}{25} = \frac{6}{5}$ (*ii*) $\frac{-6}{25} \div \frac{3}{5} = \frac{-6}{25} \times \frac{5}{3} = \frac{-6 \times 5}{25 \times 3} = \frac{-30}{75} = \frac{-2}{5}$

(*iii*) $\frac{11}{24} \div \frac{(-5)}{8} = \frac{11}{24} \times \frac{8}{(-5)} = \frac{11 \times 8}{24 \times (-5)} = \frac{88}{-120} = \frac{11}{-15} = \frac{-11}{15}$

2. *Evaluate*: $\frac{-12}{45} \div \frac{(-4)}{9}$

Sol. $\frac{-12}{45} \div \frac{(-4)}{9} = \frac{-12}{45} \times \frac{9}{-4} = \frac{3 \times 1}{5 \times 1} = \frac{3}{5}$

3. *Find the value of:*

(*i*) $\frac{8}{7} \div \left(\frac{-5}{7}\right)$ (*ii*) $0 \div \frac{6}{7}$ (*iii*) $\frac{3}{13} \div \left(\frac{-4}{65}\right)$

Sol. (*i*) $\frac{8}{7} \div \left(\frac{-5}{7}\right) = \frac{8}{7} \times \left(\frac{-7}{5}\right) = \frac{8}{7} \times \frac{-7}{5} = \frac{8}{1} \times \frac{-1}{5} = \frac{8 \times (-1)}{1 \times 5} = \frac{-8}{5} = -\frac{8}{5}$

(*ii*) $0 \div \frac{6}{7} = 0 \times \frac{7}{6} = \frac{0 \times 7}{6} = \frac{0}{6} = 0$

(*iii*) $\frac{3}{13} \div \left(\frac{-4}{65}\right) = \frac{3}{13} \times \frac{65}{-4} = \frac{3}{1} \times \frac{5}{-4} = \frac{3 \times 5}{1 \times (-4)} = \frac{15}{-4} = \frac{-15}{4}$

4. *Simplify* $\frac{3}{4} - \frac{2}{3} \div \frac{8}{9}$

Sol. $\frac{3}{4} - \frac{2}{3} \div \frac{8}{9} = \frac{3}{4} - \frac{2}{3} \times \frac{9}{8} = \frac{3}{4} - \frac{3}{4} = 0$

5. *Verify that* $\frac{15}{27} \div \frac{30}{81} \neq \frac{30}{81} \div \frac{15}{27}$

Sol. $\frac{15}{27} \div \frac{30}{81} = \frac{15}{27} \times \frac{81}{30} = \frac{3}{2}$

Again, $\frac{30}{81} \div \frac{15}{27} = \frac{30}{81} \times \frac{27}{15} = \frac{2 \times 1}{3 \times 1} = \frac{2}{3}$

As $\frac{3}{2} \neq \frac{2}{3}$

So, $\frac{15}{27} \div \frac{30}{81} \neq \frac{30}{81} \div \frac{15}{27}$ Hence verified.

Word Problems

Solved Examples

1. *The cost of* $\frac{19}{4}$ *metres of wire is* ₹ $\frac{171}{2}$. *Find the cost of one metre of the wire.*

Sol. The cost of $\frac{19}{4}$ m of wire = ₹ $\frac{171}{2}$

$\therefore$ The cost of 1 m of wire = ₹ $\left(\frac{171}{2} \div \frac{19}{4}\right)$

$= ₹ \left(\frac{171}{2} \times \frac{4}{19}\right) = ₹ \left(\frac{9 \times 2}{1 \times 1}\right) = ₹\ 18$

2. *If 16 shirts of equal size can be made out of 24 m of cloth, how much cloth is needed for making one shirt?*

Sol. Cloth required for 16 shirts of equal size = 24 m

$\therefore$ 1 shirt can be made out of 24 m ÷ 16 = $\frac{24}{16}$ m = $\frac{3}{2}$ m = $1\frac{1}{2}$ m

Hence, $1\frac{1}{2}$ m of cloth is needed for making 1 shirt.

3. *The product of two numbers is* $\frac{3}{4}$. *If one of the numbers is* $\frac{1}{8}$, *find the other rational number.*

Sol. Let the other number be x.

As the product of two numbers is $\frac{3}{4}$ and one of the numbers is $\frac{1}{8}$

$\therefore \quad \frac{1}{8} \times x = \frac{3}{4} \qquad \Rightarrow x = \frac{3}{4} \div \frac{1}{8}$

$\Rightarrow \quad x = \frac{3}{4} \times \frac{8}{1} = \frac{3 \times 2}{1 \times 1} = \frac{6}{1} = 6$

Hence, the other rational number is 6.

4. $\frac{7}{11}$ *of the total money in Hamid's bank account is* ₹ 77,000. *How much money does Hamid has in his bank account?*

Sol. Let Hamid has ₹ x in his bank account.

Now $\quad \frac{7}{11}$ of x = ₹ 77,000 $\qquad \therefore \frac{7}{11} \times x$ = ₹ 77,000

$\Rightarrow \quad x = 77{,}000 \div \frac{7}{11} = ₹\ 77{,}000 \times \frac{11}{7} = ₹\ 11{,}000 \times 11 = ₹\ 1{,}21{,}000$

So, Hamid has ₹ 1,21,000 in his bank account.

5. *A* $117\frac{1}{3}$ *m long rope is cut into equal pieces measuring* $7\frac{1}{3}$ *m each. How such small pieces are there?*

Sol. Let number of small pieces of rope made be x.

Length of each small piece is $7\frac{1}{3}$ m.

$\therefore \quad x \times 7\frac{1}{3} = 117\frac{1}{3}$ m

$\Rightarrow \quad x = 117\frac{1}{3} \div 7\frac{1}{3} = \frac{352}{3} \div \frac{22}{3} = \frac{352}{3} \times \frac{3}{22} = \frac{16 \times 1}{1 \times 1} = \frac{16}{1} = 16$

Hence, 16 small pieces are there.

6. *The product of two rational numbers is* $\frac{-14}{27}$. *If one of the numbers be* $\frac{7}{9}$, *find the other.*

Sol. Let the other number be x.

According to the question, we have

$$\frac{7}{9} \times x = \frac{-14}{27} \qquad \Rightarrow x = \frac{-14}{27} \div \frac{7}{9}$$

$$\Rightarrow \qquad x = \frac{-14}{27} \times \frac{9}{7} = \frac{-2 \times 1}{3 \times 1} = \frac{-2}{3}$$

Hence, the other rational number is $\frac{-2}{3}$.

7. *By what number should we multiply* $\frac{-15}{20}$ *so that the product may be* $\frac{-5}{7}$?

Sol. Let the required number be x.

According to the question, we have

$$\frac{-15}{20} \times x = \frac{-5}{7} \qquad \Rightarrow x = \frac{-5}{7} \div \frac{-15}{20}$$

$$\Rightarrow \qquad x = \frac{-5}{7} \times \frac{20}{-15} = \frac{-5 \times 20}{7 \times (-15)} = \frac{20}{7 \times 3} = \frac{20}{21}$$

Hence, the required number is $\frac{20}{21}$.

8. *By what number should we multiply* $\frac{-8}{13}$ *so that the product may be 24?*

Sol. Let the required number be x.

According to the question, we have

$$\therefore \qquad \frac{-8}{13} \times x = 24 \qquad \Rightarrow x = 24 \div \frac{-8}{13}$$

$$\Rightarrow \qquad x = 24 \times \frac{13}{-8} = 3 \times \frac{13}{-1} = -39$$

9. *The product of two rational numbers is* -7. *If one of the numbers is* -5, *find the other.*

Sol. Let the other number be x.

According to the question, we have

$$-5 \times x = -7 \qquad \Rightarrow x = (-7) \div (-5)$$

$$\Rightarrow \qquad x = \frac{-7}{-5} = \frac{7}{5}$$

Exercise 1.5

1. State true (T) or false (F) for each of the following statements:

(*i*) The division of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ is defined as

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr}$$ where p, q, r, s are integers and only p may be 0.

(*ii*) $\frac{4}{3} \div \frac{9}{5} = \frac{20}{27}$

(*iii*) Division of rational numbers is not commutative.

(*iv*) Division is not associative for rational numbers.

(*v*) The product of two rational numbers is $\frac{-16}{25}$. If one of them is $\frac{-4}{5}$, then the other is $\frac{3}{5}$.

2. Fill in the blanks:

(*i*) $\frac{5}{8} \div (\ldots\ldots) = \frac{5}{14}$ (*ii*) $(\ldots\ldots\ldots) \div \frac{5}{-7} = \frac{-14}{19}$

(*iii*) $(\ldots\ldots\ldots) \div (-5) = \frac{-4}{25}$ (*iv*) $(-18) \div (\ldots\ldots\ldots) = \frac{-6}{5}$

3. Divide:

(*i*) $\frac{8}{9}$ by $\frac{2}{81}$ (*ii*) $89\frac{1}{4}$ by $3\frac{1}{2}$ (*iii*) $\frac{-15}{56}$ by $\frac{5}{48}$

4. The product of two rational numbers is –35. If one of them is $\frac{5}{-7}$, find the other rational number.

5. By what number should we divide $\frac{-1}{8}$ to get $\frac{-3}{10}$?

6. The product of two rational numbers is 25. If one of them is –20, find the other.

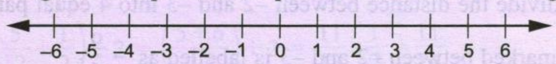
7. If 56 m long ribbon is cut into equal pieces measuring $1\frac{1}{3}$ m each. How many such pieces of ribbon are formed?

8. If 25 shirts of equal size can be made out of $37\frac{1}{2}$ m cloth, how much cloth is needed for making one shirt?

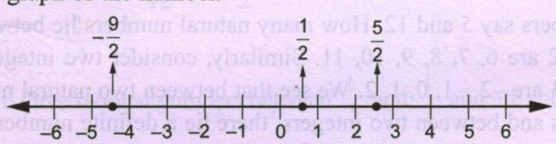
9. The product of two rational numbers is $\frac{-8}{9}$. If one of the numbers is $\frac{-4}{15}$, find the other.

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

You have already learnt in previous classes how to represent integers on a number line. To construct a number line, we first draw a straight line and divide it into equal units of length. We choose a point on the line as our base point and call this base point the **origin** and associate the number 0 with this point. Now from zero, move one mark to the right. We associate the number 1 to this point. Again, from 1, move one mark to the right and associate the number 2 to this point, etc. The positive integers correspond to the points to the right of the origin and negative integers correspond to the points to the left of the origin.



If we place a dot on the number line to indicate the location of a number, we say that we have graphed the number and that dot is the graph of the number.



On the number line given above, we indicate the position of $\frac{1}{2}, \frac{5}{2}$ and $-\frac{9}{2}$. The point on the number line which is half way between 0 and 1 has been labelled $\frac{1}{2}$, the point on the number line which is half way between 2 and 3 has been labelled $\frac{5}{2}$. The point on the number line which is half way between –4 and –5 is $-\frac{9}{2}$.

As required we can indicate the position of any rational number by locating its relation to the number shown. To indicate the position of a rational number $\frac{1}{3}$, divide the distance between 0 and 1 in three equal parts. The first, of equally spaced points, can be labelled $\frac{1}{3}$.

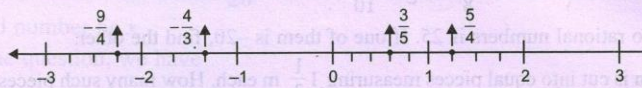
Similarly, to indicate the position of a rational number $\frac{11}{5} = 2\frac{1}{5}$, divide the distance between 2 and 3 in five equal parts. The first, of equally spaced points that divide the distance between 2 and 3 into five equal parts, can be labelled as $\frac{11}{5} = 2\frac{1}{5}$.

Let us represent each of the following rational numbers on the number line.

(*i*) $\frac{3}{5}$ (*ii*) $\frac{5}{4}$ (*iii*) $-\frac{4}{3}$ (*iv*) $-\frac{9}{4}$

Draw a straight line and divide it into equal units of length. Choose a point on the line as our base point and associate the number 0 with this point.

From zero, move to the right and mark three equally spaced points and associate positive integers 1, 2 and 3 to these points. Again from zero, move to the left and mark three equally spaced points and associate negative integers –1, –2 and –3 to these points.



(*i*) As $0 < \frac{3}{5} < 1$, so divide the distance between 0 and 1 in 5 equal parts. The third point from 0 to the right of origin and marked between 0 and 1 is labelled as $\frac{3}{5}$.

(*ii*) As $\frac{5}{4} = 1\frac{1}{4}$, divide the distance between 1 and 2 into 4 equal parts. The first point from 1 to the right of 1 and marked between 1 and 2 is labelled as $1\frac{1}{4} = \frac{5}{4}$.

(*iii*) As $-\frac{4}{3} = -1\frac{1}{3}$, so divide the distance between –1 and –2 into 3 equal parts. The first point from –1 to the left of –1 and marked between –1 and –2 is labelled as $-1\frac{1}{3} = -\frac{4}{3}$.

(*iv*) As $-\frac{9}{4} = -2\frac{1}{4}$, so divide the distance between –2 and –3 into 4 equal parts. The first point from –2 to the left of –2 and marked between –2 and –3 is labelled as $-\frac{9}{4}$.

Rational Numbers between Two Rational Numbers

Consider two natural numbers say 5 and 12. How many natural numbers lie between 5 and 12? The natural numbers between 5 and 12 are 6, 7, 8, 9, 10, 11. Similarly, consider two integers –3 and 3. The integers which lie between –3 and 3 are –2, –1, 0, 1, 2. We see that between two natural numbers, there lie a definite number of natural numbers and between two integers, there lie a definite number of integers.

Now, consider two rational numbers say 0 and 1. $\frac{1}{2}$ is a rational number which lies between 0 and 1.

But $\frac{1}{2} = \frac{2}{4} = \frac{5}{10} = \frac{10}{20} = \frac{6}{12} = \frac{20}{40} = \frac{100}{200}$, and so on.

We see that $\frac{2}{4}, \frac{5}{10}, \frac{6}{12}, \frac{20}{40}, \frac{100}{200}$, etc., all are rational numbers greater than 0 and less than 1. They lie between 0 and 1.

$\frac{1}{3}, \frac{1}{5}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{5}{10}, \frac{6}{10}, \frac{8}{10}, \frac{31}{100}, \frac{32}{100}, \frac{33}{100}$, and so on; are rational numbers between 0 and 1.

We can go on inserting more and more rational numbers between 0 and 1. So, unlike natural numbers and integers, the number of rational numbers which lie between two rational numbers is not definite. In general, there are infinitely (countless) many rational numbers between any two given rational numbers.

If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

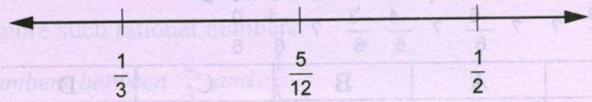
Consider two rational numbers $\frac{1}{3}$ and $\frac{1}{2}$.

Let us find three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

The mean of these two rational numbers is

$$\left(\frac{1}{3}+\frac{1}{2}\right)\div 2 = \frac{5}{6}\times\frac{1}{2} = \frac{5}{12}$$

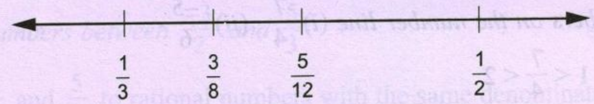
So, $\frac{5}{12}$ is a rational number which lies between $\frac{1}{3}$ and $\frac{1}{2}$.



$$\frac{1}{3} < \frac{5}{12} < \frac{1}{2}$$

Now, we find the mean of $\frac{1}{3}$ and $\frac{5}{12}$.

$$\left(\frac{1}{3}+\frac{5}{12}\right)\div 2 = \left(\frac{4+5}{12}\right)\div 2 = \frac{9}{12}\times\frac{1}{2} = \frac{9}{24} = \frac{3}{8}$$



$$\frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{1}{2}$$

Now, we find a rational number between $\frac{5}{12}$ and $\frac{1}{2}$.

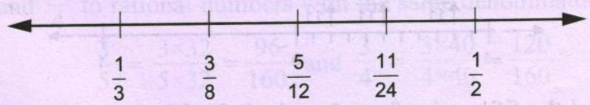
For this we find the mean of $\frac{5}{12}$ and $\frac{1}{2}$.

$$\left(\frac{5}{12}+\frac{1}{2}\right)\div 2 = \left(\frac{5+6}{12}\right)\div 2 = \frac{11}{12}\times\frac{1}{2} = \frac{11}{24}$$

So, $\frac{11}{24}$ is a rational number between $\frac{5}{12}$ and $\frac{1}{2}$.

Now, we have $\frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{11}{24} < \frac{1}{2}$

Thus, $\frac{3}{8}$, $\frac{5}{12}$ and $\frac{11}{24}$ are three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$ and are shown on the number line given below.



In this manner, we can find as many rational numbers as we want between two given rational numbers. In general, there are countably infinite numbers between any two given rational numbers.

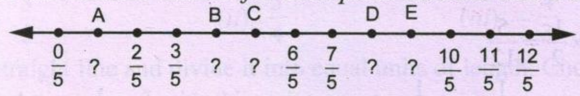
Remember

1. To find a rational number between a and b, you can add a and b and divide the sum by 2, that is $\frac{a+b}{2}$ lies between a and b.
2. There are countless rational numbers between any two given rational numbers.

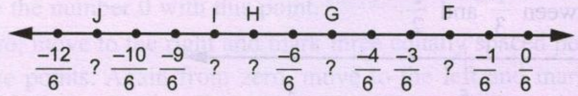
Solved Examples

1. *Write the rational number for each point labelled with a letter.*

(*i*)



(*ii*)



Sol. (*i*)

Points	A	B	C	D	E
Rational Numbers	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{5}{5}=1$	$\frac{8}{5}$	$\frac{9}{5}$

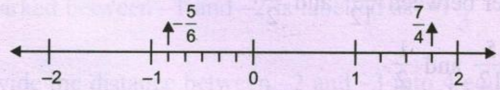
(*ii*)

Points	F	G	H	I	J
Rational Numbers	$\frac{-2}{6}$	$\frac{-5}{6}$	$\frac{-7}{6}$	$\frac{-8}{6}$	$\frac{-11}{6}$

2. *Represent these numbers on the number line* (*i*) $\frac{7}{4}$ (*ii*) $\frac{-5}{6}$.

Sol. (*i*) As $\frac{7}{4} = 1\frac{3}{4}$, so $1 < \frac{7}{4} < 2$

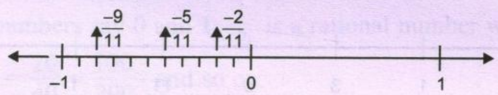
Divide the distance between 1 and 2 in 4 equal parts and label the third point from 1 to the right of 1 marked between 1 and 2 as $\frac{7}{4}$.



(*ii*) As $-1 < \frac{-5}{6} < 0$, so divide the distance between −1 and 0 into 6 equal parts and label the fifth point from 0 to the left of 0 marked between −1 and 0 as $\frac{-5}{6}$ (see above).

3. *Represent* $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ *on the number line.*

Sol. As $-1 < \frac{-2}{11} < 0$, so divide the distance between −1 and 0 into 11 equal parts and label the second point from 0 to the left of 0 marked between −1 and 0 as $\frac{-2}{11}$.



As $-1 < \frac{-5}{11} < 0$, label the fifth point from 0 to the left of 0 marked between −1 and 0 as $\frac{-5}{11}$.

As $-1 < \frac{-9}{11} < 0$, label the ninth point from 0 to the left of 0 marked between −1 and 0 as $\frac{-9}{11}$.

The numbers $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ are shown on the number line (see above).

4. *Write five rational numbers which are less than* 2.

Sol. There are countless rational numbers which are less than 2. We give below five rational numbers which are less than 2.

$$\frac{-1}{4}, 0, \frac{1}{2}, 1, \frac{3}{2}$$

5. *Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.*

Sol. We first convert $\frac{-2}{5}$ and $\frac{1}{2}$ to rational numbers with the same denominators.

We can write $\frac{-2}{5} = \frac{-2\times4}{5\times4} = \frac{-8}{20}$ and $\frac{1}{2} = \frac{1\times10}{2\times10} = \frac{10}{20}$

Thus, we have $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{2}{20}$ as the ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

There can be many more such rational numbers.

6. *Find five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$.*

Sol. We first convert $\frac{2}{3}$ and $\frac{4}{5}$ to rational numbers with the same denominators.

We have $\frac{2}{3} = \frac{2\times20}{3\times20} = \frac{40}{60}$ and $\frac{4}{5} = \frac{4\times12}{5\times12} = \frac{48}{60}$

Now, the five rational numbers $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$ are all between $\frac{2}{3}$ and $\frac{4}{5}$.

7. *Find five rational numbers between $\frac{-3}{2}$ and $\frac{5}{3}$.*

Sol. We first convert $\frac{-3}{2}$ and $\frac{5}{3}$ to rational numbers with the same denominators.

We can write $\frac{-3}{2} = \frac{-3\times3}{2\times3} = \frac{-9}{6}$ and $\frac{5}{3} = \frac{5\times2}{3\times2} = \frac{10}{6}$

We have $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$ are the five rational numbers between $\frac{-3}{2}$ and $\frac{5}{3}$.

8. *Find five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.*

Sol. We first convert $\frac{1}{4}$ and $\frac{1}{2}$ to rational numbers with the same denominators.

We can write $\frac{1}{4} = \frac{1\times8}{4\times8} = \frac{8}{32}$ and $\frac{1}{2} = \frac{1\times16}{2\times16} = \frac{16}{32}$

We have $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$ are the five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

9. *Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.*

Sol. We first convert $\frac{3}{5}$ and $\frac{3}{4}$ to rational numbers with the same denominators.

We can write $\frac{3}{5} = \frac{3\times32}{5\times32} = \frac{96}{160}$ and $\frac{3}{4} = \frac{3\times40}{4\times40} = \frac{120}{160}$

Now, the ten rational numbers

$\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$, are all lie between $\frac{3}{5}$ and $\frac{3}{4}$.

COMPARISON OF RATIONAL NUMBERS

We know how to compare two integers. We know that (*i*) Every positive integer is greater than 0 (*ii*) Every negative integer is less than 0 (*iii*) Every positive integer is greater than every negative integer.

Similarly, we have the following facts about how to compare the rational numbers:

(*i*) Every positive rational number is greater than 0.

(*ii*) Every negative rational number is less than 0.

(*iii*) Every positive rational number is greater than every negative rational number.

(*iv*) Every rational number, represented by a point on the number line, is greater than every rational number represented by a point on the left and is less than every rational number represented by a point on its right.

In order to compare any two rational numbers, use the following steps:

1. Obtain the given rational numbers and write them, so that their denominators are positive.
2. Find the LCM of the denominators.
3. Make the denominators same using LCM, that is, express each rational number with the LCM as common denominator.
4. Compare the numerators of rational numbers obtained, having greater numerator, is the greater rational number. Give them proper inequality sign.

Solved Examples

1. *Which of the two rational numbers* $\frac{4}{7}$ *and* $\frac{-3}{5}$ *is greater?*

Sol. We see that $\frac{4}{7}$ is a positive rational number and $\frac{-3}{5}$ is a negative rational number. As positive rational numbers are always greater than the negative numbers, so $\frac{4}{7} > \frac{-3}{5}$.

2. *Which of the two rational numbers* $\frac{-2}{7}$ *and* $\frac{3}{-5}$ *is greater?*

Sol. Let us first write each of the given rational numbers with positive denominator.

Denominator of $\frac{-2}{7}$ is positive. Denominator of $\frac{3}{-5}$ is negative.

So, $$\frac{3}{-5} = \frac{3\times(-1)}{(-5)\times(-1)} = \frac{-3}{5}$$

[Multiplying the numerator and denominator by (–1)]

Now, LCM of 7 and 5 is 35.

Writing the rational numbers $\frac{-2}{7}$ and $\frac{-3}{5}$ so that they have a common denominator 35 as follows.

$$\frac{-2}{7} = \frac{-2}{7}\times\frac{5}{5} = \frac{(-2)\times5}{7\times5} = \frac{-10}{35} \text{ and } \frac{-3}{5} = \frac{-3}{5}\times\frac{7}{7} = \frac{(-3)\times7}{5\times7} = \frac{-21}{35}$$

Now, comparing the numerators of $\frac{-10}{35}$ and $\frac{-21}{35}$, we see that $-10 > -21$

$$\therefore \frac{-10}{35} > \frac{-21}{35} \Rightarrow \frac{-2}{7} > \frac{-3}{5} \Rightarrow \frac{-2}{7} > \frac{3}{-5}$$

Hence, $\frac{-2}{7}$ is greater than $\frac{3}{-5}$.

3. *Arrange* $\frac{-2}{3}, \frac{5}{-9}, \frac{-8}{15}$ *in ascending order.*

Sol. First, we write the given rational numbers with positive denominators, so we have

$$\frac{-2}{3}, \frac{-5}{9}, \frac{-8}{15}$$

LCM of denominators 3, 9, 15 is 45.

Writing the given fractions with denominator 45 (LCM), we get

$$\frac{-2}{3} = \frac{-2\times15}{3\times15} = \frac{-30}{45}, \frac{-5}{9} = \frac{-5\times5}{9\times5} = \frac{-25}{45}, \frac{-8}{15} = \frac{-8}{15}\times\frac{3}{3} = \frac{-24}{45}$$

Now, comparing the numerators of these rational numbers, we get

$$-30 < -25 < -24$$

$$\therefore \quad \frac{-30}{45} < \frac{-25}{45} < \frac{-24}{45}$$

$$\Rightarrow \quad \frac{-2}{3} < \frac{-5}{9} < \frac{-8}{15}$$

$$\Rightarrow \quad \frac{-2}{3} < \frac{5}{-9} < \frac{-8}{15}$$

4. *Arrange* $\frac{2}{-5}, \frac{-3}{7}, \frac{9}{35}$ *in descending order.*

Sol. First, we write the given rational numbers with positive denominators, so we get

$$\frac{-2}{5}, \frac{-3}{7}, \frac{9}{35}$$

LCM of denominators 5, 7, 35 is 35.

Now, writing each rational number with denominator 35 (LCM), we get

$$\frac{-2}{5} = \frac{-2}{5} \times \frac{7}{7} = \frac{-14}{35}, \quad \frac{-3}{7} = \frac{-3}{7} \times \frac{5}{5} = \frac{-15}{35} \text{ and } \frac{9}{35} = \frac{9}{35}$$

Comparing the numerators of these rational numbers, and writing them in descending order, we get

$$9 > -14 > -15$$

$$\therefore \quad \frac{9}{35} > \frac{-14}{35} > \frac{-15}{35}$$

$$\Rightarrow \quad \frac{9}{35} > \frac{-2}{5} > \frac{-3}{7} \Rightarrow \frac{9}{35} > \frac{2}{-5} > \frac{-3}{7}$$

Miscellaneous Solved Examples

There are four options (Q1 to Q3) out of which only one is correct. Choose the correct answer.

1. *Between any two rational numbers, there lie*

(*a*) *One rational number* (*b*) *Finite rational numbers*

(*c*) *Countless rational numbers* (*d*) *None of these*

Sol. Between any two rational numbers, there lie countless rational numbers.

Hence, (*c*) is the correct answer.

2. *Which one of the following is the rational number between* $\frac{6}{7}$ *and* $\frac{7}{8}$?

(*a*) $\frac{3}{4}$ (*b*) $\frac{99}{112}$ (*c*) $\frac{95}{112}$ (*d*) $\frac{97}{112}$

Sol. $$\frac{\frac{6}{7}+\frac{7}{8}}{2} = \frac{\frac{6\times8+7\times7}{7\times8}}{2} = \frac{48+49}{2\times7\times8} = \frac{97}{112}$$

Hence, $\frac{97}{112}$ is a rational number between $\frac{6}{7}$ and $\frac{7}{8}$.

Hence, (*d*) is the correct answer.

3. *The correct symbol out of* >, =, < *to fill the blank space in* $\frac{3}{11} \ldots \frac{-4}{11}$ *is*

(*a*) < (*b*) = (*c*) > (*d*) ≥

Sol. Here, the two rational numbers have the same positive denominator, and $3 > -4$.

So, the correct symbol to be filled in the blank space is >.

Hence, (*c*) is the correct answer.

4. *Replace □ with <, >, or = to make each statement a true statement.*

(*i*) $\frac{1}{5} \square \frac{1}{4}$ (*ii*) $\frac{1}{5} \square \frac{1}{7}$ (*iii*) $-4\frac{1}{4} \square -4$

(*iv*) $12\frac{1}{2} \square 12\frac{3}{4}$ (*v*) $\frac{-5}{7} \square \frac{-2}{7}$ (*vi*) $\frac{-9}{20} \square \frac{-11}{20}$

Sol. (*i*) < (*ii*) > (*iii*) < (*iv*) < (*v*) < (*vi*) >

5. *Give a rational number between* $\frac{5}{6}$ *and* $\frac{6}{7}$.

Sol. We know that if *a* and *b* are two rational numbers, then $\frac{a+b}{2}$ is rational number between *a* and *b*.

The rational number $\dfrac{\frac{5}{6}+\frac{6}{7}}{2} = \dfrac{\frac{5\times7+6\times6}{6\times7}}{2} = \dfrac{5\times7+6\times6}{2\times6\times7} = \dfrac{71}{84}$ lies between $\frac{5}{6}$ and $\frac{6}{7}$.

6. *Insert two rational numbers between* $\frac{4}{5}$ *and* $\frac{6}{7}$.

Sol. $\dfrac{\frac{4}{5}+\frac{6}{7}}{2} = \dfrac{\frac{4\times7+6\times5}{5\times7}}{2} = \dfrac{4\times7+6\times5}{2\times5\times7} = \dfrac{28+30}{70} = \dfrac{58}{70} = \dfrac{29}{35}$

So, we have $\frac{4}{5} < \frac{29}{35} < \frac{6}{7}$

Now, we find a rational number between $\frac{4}{5}$ and $\frac{29}{35}$.

$$\dfrac{\frac{4}{5}+\frac{29}{35}}{2} = \dfrac{\frac{4\times7+29}{35}}{2} = \dfrac{28+29}{35\times2} = \dfrac{57}{70}$$

$\frac{57}{70}$ lies between $\frac{4}{5}$ and $\frac{29}{35}$ and hence between $\frac{4}{5}$ and $\frac{6}{7}$.

i.e., $\frac{4}{5} < \frac{57}{70} < \frac{29}{35} < \frac{6}{7}$

Hence, the two rational numbers $\frac{57}{70}$ and $\frac{29}{35}$ lie between $\frac{4}{5}$ and $\frac{6}{7}$.

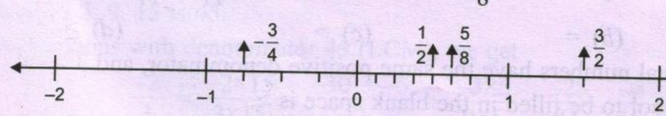
7. *Represent the rational numbers* $\frac{1}{2}, \frac{-3}{4}, \frac{3}{2}$ *and* $\frac{5}{8}$ *on the number line.*

Sol. To indicate $\frac{1}{2}$ on the number line, divide the distance between 0 and 1 into two equal parts. The point which divides the distance between two equal parts is labelled as $\frac{1}{2}$.

Again, to indicate $\frac{-3}{4}$ on the number line, divide the distance between 0 and –1 into four equal parts. The third point from 0 to the left of origin and marked between 0 and 1 is labelled as $\frac{-3}{4}$.

Now, $\frac{3}{2} = 1\frac{1}{2}$. The point on the number line which is half way between 1 and 2 is labelled $\frac{3}{2}$.

As $0 < \frac{5}{8} < 1$, so divide the distance between 0 and 1 in 8 equal parts. The fifth point from 0 to the right of origin and marked between 0 and 1 is labelled as $\frac{5}{8}$.



8. *Which of the numbers given in each pair is greater?*

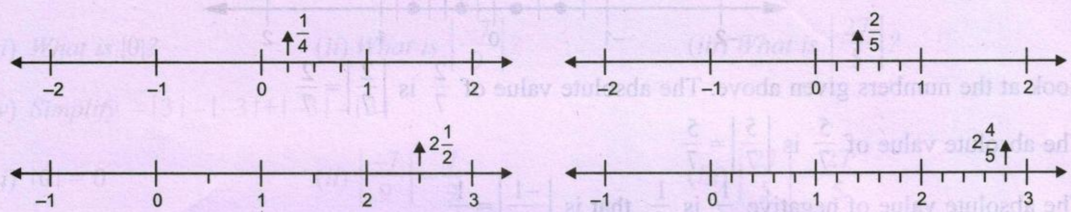
(*i*) $\frac{3}{-4}, \frac{-5}{6}$ (*ii*) $\frac{3}{7}, \frac{-4}{9}$ (*iii*) $\frac{-7}{12}, \frac{5}{-8}$ (*iv*) $\frac{(-3)}{7}, \frac{6}{(-13)}$

(*v*) $0, \frac{-4}{9}$

Sol. (*i*) $\frac{3}{-4} > \frac{-5}{6}$ (*ii*) $\frac{3}{7} > \frac{-4}{9}$ (*iii*) $\frac{-7}{12} > \frac{5}{-8}$ (*iv*) $\frac{-3}{7} > \frac{6}{-13}$ (*v*) $0 > \frac{-4}{9}$

9. *Represent* $\frac{1}{4}, \frac{2}{5}, 2\frac{1}{2}$ *and* $2\frac{4}{5}$ *on number lines.*

Sol.



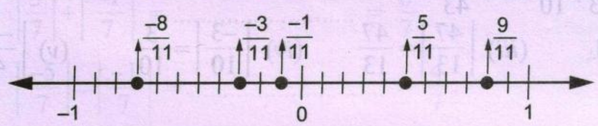
10. (*i*) *Arrange the rational numbers* $\frac{3}{7}, \frac{-5}{9}, \frac{7}{-11}, \frac{6}{1}, \frac{0}{1}$ *and* $\frac{9}{3}$ *in ascending order.*

(*ii*) *Find three rational numbers between* $\frac{2}{3}$ *and* $\frac{5}{6}$.

(*iii*) *Represent* $\frac{-8}{11}, \frac{-3}{11}, \frac{-1}{11}, \frac{5}{11}$ *and* $\frac{9}{11}$ *on a number line.*

Sol. (*i*) $\frac{-7}{11} < \frac{-5}{9} < \frac{0}{1} < \frac{3}{7} < \frac{9}{3} < \frac{6}{1}$ (*ii*) $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}$

(*iii*)



Exercise 1.6

1. Represent the following rational numbers on the number line.

(*i*) $-\frac{2}{3}$ (*ii*) $\frac{11}{4}$ (*iii*) $-\frac{13}{5}$ (*iv*) $-\frac{19}{6}$

2. Write three rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.

3. Fill in the boxes with >, < or =.

(*i*) $\frac{-3}{5} \square \frac{3}{5}$ (*ii*) $\frac{-5}{7} \square \frac{10}{-14}$ (*iii*) $\left|\frac{-7}{9}\right| \square 0$ (*iv*) $\left|\frac{-3}{4}\right| \square \frac{-2}{3}$

4. Which of the two given rational numbers is greater?

(*i*) $\frac{3}{8}$ or $\frac{8}{9}$ (*ii*) $-\frac{1}{4}$ or $\frac{-3}{5}$ (*iii*) $\frac{-4}{-9}$ or $\frac{6}{7}$

5. Arrange in ascending order.

(*i*) $\frac{3}{4}, \frac{-2}{5}, \frac{7}{8}, \frac{5}{6}$ (*ii*) $-\frac{3}{5}, \frac{-7}{5}, 0, \frac{-1}{7}$

6. Arrange in descending order.

(*i*) $\frac{5}{12}, \frac{-1}{4}, \frac{2}{3}, \frac{8}{9}$ (*ii*) $-\frac{1}{3}, \frac{5}{6}, \frac{-7}{8}, \frac{5}{12}$

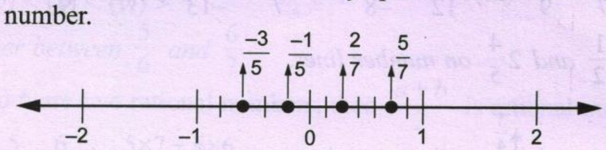
7. Write three rational numbers between

(*i*) –1 and 1 (*ii*) –4 and –3 (*iii*) $\frac{-3}{5}$ and $\frac{-2}{3}$

ABSOLUTE VALUE OF A RATIONAL NUMBER

Rational numbers are the numbers that can be written as the fraction of two integers and absolute value is the distance a number is away from 0. Distance is always positive.

As distance is always positive, so absolute value is always positive. Absolute value of a number is written as two bars | | around the number.



Look at the numbers given above. The absolute value of $\frac{2}{7}$ is $\left|\frac{2}{7}\right| = \frac{2}{7}$

The absolute value of $\frac{5}{7}$ is $\left|\frac{5}{7}\right| = \frac{5}{7}$

The absolute value of negative $\frac{1}{5}$ is $\frac{1}{5}$, that is $\left|\frac{-1}{5}\right| = \frac{1}{5}$

The absolute value of negative $\frac{3}{5}$ is $\frac{3}{5}$, that is $\left|\frac{-3}{5}\right| = \frac{3}{5}$

Remember

Absolute value shows the distance of a number from zero. Absolute value of a number is always positive.

Let us find the absolute value of 5, –21, $\frac{47}{13}, \frac{-3}{10}$ and $\frac{-21}{43}$.

(*i*) $|5| = 5$ (*ii*) $|-21| = 21$ (*iii*) $\left|\frac{47}{13}\right| = \frac{47}{13}$ (*iv*) $\left|\frac{-3}{10}\right| = \frac{3}{10}$ (*v*) $\left|\frac{-21}{43}\right| = \frac{21}{43}$

Solved Examples

1. *Write the absolute value of each of the following rational numbers.*

(*i*) $-\frac{9}{23}$ (*ii*) $\frac{-3}{11}$ (*iii*) $\frac{7}{15}$ (*iv*) $\frac{-14}{5}$

Sol. (*i*) $\left|\frac{-9}{23}\right| = \frac{9}{23}$ (*ii*) $\left|\frac{-3}{11}\right| = \frac{3}{11}$ (*iii*) $\left|\frac{7}{15}\right| = \frac{7}{15}$ (*iv*) $\left|\frac{-14}{5}\right| = \frac{14}{5}$

2. *Find* $\left|\frac{2}{7} - \frac{3}{7}\right| + \left|\frac{2}{5} \times \frac{5}{8}\right|$

Sol. We have $\left|\frac{2}{7} - \frac{3}{7}\right| + \left|\frac{2}{5} \times \frac{5}{8}\right| = \left|\frac{2-3}{7}\right| + \left|\frac{1}{4}\right| = \left|\frac{-1}{7}\right| + \left|\frac{1}{4}\right|$

$= \frac{1}{7} + \frac{1}{4} = \frac{1 \times 4}{7 \times 4} + \frac{1 \times 7}{4 \times 7}$ [LCM of 4 and 7 is 28]

$= \frac{4}{28} + \frac{7}{28} = \frac{4+7}{28} = \frac{11}{28}$

3. *Verify that* $|x+y| < |x| + |y|$ *if* $x = \frac{5}{12}, y = \frac{-7}{18}$

Sol. $|x+y| = \left|\frac{5}{12} + \frac{-7}{18}\right| = \left|\frac{5 \times 3 + (-7) \times 2}{36}\right| = \left|\frac{15-14}{36}\right| = \left|\frac{1}{36}\right| = \frac{1}{36}$

and $|x| + |y| = \left|\frac{5}{12}\right| + \left|\frac{-7}{18}\right| = \frac{5}{12} + \frac{7}{18} = \frac{(5 \times 3) + (7 \times 2)}{36} = \frac{15+14}{36} = \frac{29}{36}$

As $\frac{1}{36} < \frac{29}{36}$, so $|x+y| < |x| + |y|$ is verified.

4. *Verify that* $|x \times y| = |x| \times |y|$ *if* $x = \frac{-5}{3}, y = \frac{7}{9}$

Sol. $|x \times y| = \left|\frac{-5}{3} \times \frac{7}{9}\right| = \left|\frac{-5\times7}{3\times9}\right| = \left|\frac{-35}{27}\right| = \frac{35}{27}$

and $|x| \times |y| = \left|\frac{-5}{3}\right| \times \left|\frac{7}{9}\right| = \frac{5}{3} \times \frac{7}{9} = \frac{5\times7}{3\times9} = \frac{35}{27}$

Hence, $|x \times y| = |x| \times |y|$ is verified.

5. (*i*) *What is* $|0|$? (*ii*) *What is* $\left|\frac{-7}{9}\right|$? (*iii*) *What is* $\left|\frac{27}{5}\right|$?

(*iv*) *Simplify* $-|3| - |-3| + |-3| - |3|$

Sol. (*i*) $|0| = 0$ (*ii*) $\left|\frac{-7}{9}\right| = \frac{7}{9}$ (*iii*) $\left|\frac{27}{5}\right| = \frac{27}{5}$

(*iv*) $-|3| - |-3| + |-3| - |3| = -3 - 3 + 3 - 3 = -6$

6. *Fill in the blanks.*

(*i*) *All rational numbers* (*only whole numbers*) *whose absolute value is less than 2 are ....*

(*ii*) $|x|$ = ... *for a positive rational number x and* $|x|$ = ... *for a negative rational number x.*

(*iii*) $\left|\frac{5}{7}\right| + \left|\frac{-1}{7}\right|$ = ... $= \frac{6}{7}$.

(*iv*) $\left|\frac{-5}{7}\right| - \left|\frac{-1}{7}\right|$ = ... $= \frac{4}{7}$.

(*v*) $\left|\frac{-11}{13}\right|\left|\frac{6}{7}\right|$ = ... $= \frac{66}{91}$.

Sol. (*i*) −1, 0, 1 (*ii*) *x*, (*iii*) $\frac{5}{7} + \frac{1}{7}$ (*iv*) $\frac{5}{7} - \frac{1}{7}$ (*v*) $\frac{11}{13} \times \frac{6}{7}$

7. *A factory owner recycled* $\frac{7}{8}$ *of aluminium cans. Is the rational number expressing the amount of aluminium cans recycled more than* $\frac{1}{2}$ *or less than* $\frac{1}{2}$? [V. Imp]

Sol. We have to compare rational numbers $\frac{7}{8}$ and $\frac{1}{2}$

LCM of denominators, 8 and 2 is 8.

Now, we write the numbers $\frac{7}{8}$ and $\frac{1}{2}$ so that they have a common denominator 8.

$$\frac{7}{8} = \frac{7}{8} \text{ and } \frac{1}{2} = \frac{1\times4}{2\times4} = \frac{4}{8}$$

Now, compare the numerators 7 and 4. Since 7 > 4, so $\frac{7}{8} > \frac{4}{8}$ or $\frac{7}{8} > \frac{1}{2}$

Hence, amount of aluminium cans recycled $\left(\frac{7}{8}\right)$ is more than $\frac{1}{2}$.

Exercise 1.7

1. State true (T) or false (F) for each of the following statements:

(*i*) If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left|\frac{a}{b}+\frac{c}{d}\right| \le \left|\frac{a}{b}\right|+\left|\frac{c}{d}\right|$ and $\left|\frac{a}{b}\times\frac{c}{d}\right| = \left|\frac{a}{b}\right|\times\left|\frac{c}{d}\right|$.

(*ii*) $\left|\frac{-2}{3}\right|+\left|\frac{7}{9}\right|-\left|\frac{-5}{6}\right|=\frac{11}{18}$.

(*iii*) The absolute value of zero is equal to 0, *i.e.*, $|0| = 0$ because, zero is zero units from 0.

(*iv*) The multiplicative inverse of $\left|\frac{-3}{5}\right|$ is $\frac{5}{3}$.

2. Verify that:

(*i*) $|x+y|<|x|+|y|$ if $x=\frac{4}{7}$, $y=\frac{-5}{21}$

(*ii*) $|x+y|=|x|+|y|$ if $x=\frac{-7}{15}$, $y=\frac{-9}{25}$

3. Verify that $|x\times y|=|x|\times|y|$

(*i*) $x=\frac{-3}{5}$, $y=\frac{11}{20}$

(*ii*) $x=\frac{-4}{7}$, $y=\frac{-9}{13}$

4. Evaluate:

(*i*) $\left|\frac{11}{8}\right|-\left|\frac{-1}{4}\right|$

(*ii*) $-7-\left|\frac{-2}{7}\right|$

(*iii*) $13-\left|-\frac{1}{2}\right|+|-5|-\left|\frac{7}{8}\right|$

ACTIVITIES

1. Arrange numbers in ascending order (3/5) (−2) (1 1/4) (7)

◯ < ◯ < ◯ < ◯

2. Keeping in mind if a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a<\frac{a+b}{2}<b$, find four rational numbers between 1 and 2.

Points to Remember

1. A rational number is a number that can be written in the form $\frac{p}{q}$ where p and q are integers and $q \ne 0$. Most numbers we use in day to day life are rational numbers. Set of rational numbers is denoted by Q.

2. A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1, that is, if the HCF of p and q is 1, *i.e.*, p and q are relatively prime.

3. The operation of addition and multiplication for rational numbers are (*i*) commutative (*ii*) associative.

4. If $\frac{p}{q}$ and $\frac{r}{s}$, where $q \ne 0$, $s \ne 0$ are any two rational numbers then the sum, difference and product of these rational numbers is also a rational number, so the rational numbers are closed under operation of addition, subtraction and multiplication.

5. 0 is the additive identity for rational numbers.

6. 1 is the multiplicative identity for rational numbers.

7. The additive inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and the additive inverse of $\frac{-p}{q}$ is $\frac{p}{q}$.

8. If $\frac{a}{b} \times \frac{c}{d} = 1$, then $\frac{c}{d}$ is the reciprocal or multiplicative inverse of $\frac{a}{b}$, and vice versa.

9. Distributive property of multiplication over addition and also over subtraction.
If a, b, c are rational numbers, then (*i*) $a(b + c) = ab + ac$ (*ii*) $a(b - c) = ab - ac$.

10. We can represent rational numbers on the number line.

11. There are infinite number of rational numbers between any two rational numbers.
If a and b are two rational numbers, then $a < \frac{a+b}{2} < b$. This method is also called the arithmetic mean method.

CHAPTER TEST

I. Answer the following questions:

1. Is 0.9 a rational number?
2. Is it correct that any integer is a rational number?
3. What is the additive identity for rational numbers?
4. What is the multiplicative identity for rational numbers?
5. Additive inverse of $\frac{-a}{b}$ is $\frac{a}{b}$. Is it a correct statement?
6. Express $\frac{14}{38}$ in its lowest term.
7. Express $\frac{-8}{28}$ in standard form.
8. Arrange $\frac{-7}{10}$, $\frac{5}{-8}$ and $\frac{2}{-3}$ in ascending order.
9. Give five rational numbers between 3 and 4.
10. Simplify: $\left(\frac{7}{3} \div \frac{10}{9}\right) \times \frac{5}{11}$.
11. What should be added to $\frac{-5}{11}$ to get $\frac{3}{11}$?
12. By what number should we multiply $\frac{-15}{28}$ so that product may be $\frac{-5}{7}$?

II. State true (T) or false (F) for each of the following statements.

13. (*i*) When you add or subtract two rational numbers, you always get another rational number.
(*ii*) When you multiply two rational numbers, you always get another rational number.
(*iii*) When you divide two rational numbers, you always get another rational number provided you do not divide by zero.
(*iv*) All natural numbers, whole numbers, fractions and integers are included in rational numbers.
(*v*) 0 is not a rational number.
(*vi*) A rational number which has both, the numerator and denominator, either positive or negative is a positive rational number.
(*vii*) $\frac{-71}{-91}$ is a positive rational number.
(*viii*) A rational number, with either the numerator or the denominator negative is a negative rational number.
(*ix*) $\frac{17}{-29}$ is a negative rational number.
(*x*) If $\frac{4}{5}$ and $\frac{8}{10}$ are two rational numbers, then $\frac{4}{5} = \frac{8}{10} \Longleftrightarrow (4\times10) = (5\times8)$
(*xi*) For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, exactly one of the following is true
(*a*) $\frac{a}{b} > \frac{c}{d}$ (*b*) $\frac{a}{b} = \frac{c}{d}$ (*c*) $\frac{a}{b} < \frac{c}{d}$
(*xii*) Rearrange suitably and find, if the sum calculated is correct $\frac{2}{5} + \frac{-1}{7} + \frac{-3}{5} + \frac{3}{7} = \frac{3}{35}$

III. Fill in the blanks:

14. (*i*) The reciprocal of a positive rational number is ....
(*ii*) The rational number that are equal to their reciprocal are .... and ....
(*iii*) $-|5|+|-5|-|-5|$ = ...
(*iv*) The multiplicative inverse of $\frac{-3}{7}$ is ....
(*v*) $\frac{5}{7} \times \frac{21}{15} \div \frac{8}{7}$ = ...

(vi) $\frac{5}{7}+\frac{1}{14}-\frac{1}{21}=$ .... (vii) $\frac{3}{5}\times\left(\frac{8}{15}-\frac{1}{3}\right)=$ ....

IV. Do as directed:

15. (i) Give five examples of non-integer rational numbers.

(ii) Write five rational numbers which happen to be integers.

(iii) Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

16. Verify that:

(i) $\frac{3}{5}+\left(\frac{-1}{7}\right)=\left(\frac{-1}{7}\right)+\frac{3}{5}$

(ii) $\frac{9}{25}\times\frac{15}{12}=\frac{15}{12}\times\frac{9}{25}$ (iii) $\left(\frac{1}{5}+\frac{1}{9}\right)+\frac{2}{7}=\frac{1}{5}+\left(\frac{1}{9}+\frac{2}{7}\right)$

(iv) $\left(\frac{5}{11}\times\frac{77}{65}\right)\times\frac{3}{7}=\frac{5}{11}\times\left(\frac{77}{65}\times\frac{3}{7}\right)$ (v) $\frac{2}{7}\times\left(\frac{5}{8}-\frac{4}{9}\right)=\frac{2}{7}\times\frac{5}{8}-\frac{2}{7}\times\frac{4}{9}$

17. Tell the property used in each of the following statements:

(i) $\frac{6}{11}\times 0=0$ (ii) $\frac{-5}{13}\times 1=1\times\frac{-5}{13}=\frac{-5}{13}$

(iii) $\frac{4}{7}\times\frac{7}{4}=\frac{7}{4}\times\frac{4}{7}=1$ (iv) $\frac{3}{5}\times\left(\frac{1}{8}+\frac{5}{9}\right)=\frac{3}{5}\times\frac{1}{8}+\frac{3}{5}\times\frac{5}{9}$

(v) $\frac{2}{5}\times\left(\frac{3}{7}\times\frac{14}{9}\right)=\left(\frac{2}{5}\times\frac{3}{7}\right)\times\frac{14}{9}$

18. Verify for any rational numbers x, y and z.

(i) $x+(y+z)=(x+y)+z$ (ii) $x\times(y\times z)=(x\times y)\times z$

by taking $x=\frac{3}{5}$, $y=\frac{1}{4}$ and $z=\frac{5}{9}$

19. Simplify by applying suitable property:

(i) $\frac{3}{7}\times\left(\frac{5}{9}+\frac{4}{15}\right)$ (ii) $\frac{5}{7}\times\left(\frac{-14}{15}+0\right)$

V. Choose the correct answers.

20. What should be added to $\frac{4}{5}$ to get $\frac{49}{30}$?

(a) $\frac{5}{7}$ (b) $\frac{5}{6}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

21. By which rational number $\frac{-3}{5}$ be divided to get $\frac{-2}{3}$?

(a) $\frac{7}{10}$ (b) $\frac{11}{10}$ (c) $\frac{9}{10}$ (d) $\frac{10}{9}$

22. The multiplicative inverse of $\frac{3}{28}\times\left(\frac{-7}{12}\right)$ is

(a) –12 (b) –15 (c) –16 (d) –13

23. If we divide $\frac{21}{5}$ by the sum of $\frac{1}{5}$ and $\frac{2}{25}$, we get the quotient as

(a) 12 (b) 15 (c) 18 (d) 20

24. If we divide the difference of $\frac{1}{3}$ and $\frac{1}{5}$ by the sum of $\frac{1}{2}$ and $\frac{1}{5}$, we get

(a) $\frac{4}{17}$ (b) $\frac{4}{21}$ (c) $\frac{4}{25}$ (d) $\frac{4}{27}$

25. By which number $\frac{-7}{25}$ should be divide to get $\frac{-1}{15}$?

(a) $\frac{21}{5}$ (b) $\frac{21}{7}$ (c) $\frac{21}{19}$ (d) $\frac{21}{13}$